FRACTIONS

FRACTION

A fraction is a number representing a part of a whole.

It is written as $\frac{a}{b}$ where a is the numberator and b is the denominator. We can also say that $\frac{a}{b}$ = a ÷ b.

Examples of fractions are $\frac{2}{7}, \frac{5}{9}, \frac{6}{13}$ etc.

Fraction as a Part of a Whole :

If a pizza is divided into four equal parts, each part is represented as 1 part out of 4 equal parts and is written as $\frac{1}{4}$. The part that is considered (in this case, 1) is called the **numerator**. The number of parts into which the whole is divided (in this case, 4) is called the **denominator**.

Ex. What fraction of a day is 9 hours?

Sol. One day = 24 hours

$\therefore$ 9 hours out of = $\frac{9}{24}$

i.e., 9 hours out o f 24 hours. Hence, 9 hours is $\frac{9}{24}$ parts of a day.

NOTE:

Since time is in hours, so it is necessary to convert the two quantities, day and time in the same unit.

FRACTION AS A DIVISION

A fraction can be used to represent a division sum. For example, if one pastry has to be shared equally between two friends, the corresponding division sum would be 1 ÷ 2. We can represent this in fraction as $\frac{1}{2}$. Hence, we would say that pastry share by each friend is $\frac{1}{2}$ of the pastry.

While expressing a division sum as a fraction, the dividend is written as the numerator and the divisor is written as the denominator. Similarly, a fraction can be represented as a division sum. In that case, the numerator is written as the dividend and the denominator as the divisor.

TYPES OF FRACTION

(i) Proper Fractions

A proper fraction is a fraction in which the numerator is smaller than the denominator.

Examples of proper fractions are $\frac{2}{7}, \frac{5}{8}, \frac{12}{17}$ ..... , etc.

(ii) Improper Fractions

An improper fraction is a fraction in which the numerator is greater than the denominator.

$\frac{7}{2}, \frac{8}{5}, \frac{17}{12}$ are all improper fractions.

(iii) Mixed Fractions

A fraction like $\frac{7}{2}$ can also be expressed as $3\frac{1}{2}$, or $\frac{8}{5}$ can be written as $1\frac{3}{5}$. So, numbers of the form $3\frac{1}{2}$, $1\frac{3}{5}$, etc., are called mixed numbers or mixed fractions.

(iv) Unit fractions : Any fraction with 1 as the numerator is called a unit fraction, Thus, $\frac{1}{3}$ is called a unit fraction.

CONVERSION

CONVERSION OF MIXED FRACTIONS INTO IMPROPER FRACTIONS

$$\text{Improper fraction} = \frac{(\text{Whole no.} \times \text{denominator}) + \text{N}}{\text{Denominator}}$$

where N = Numerator

Equivalent Fraction :

Consider the following fractions.

First one is $\frac{1}{2}$, second one is $\frac{2}{4}$, and the third one is $\frac{3}{6}$. Though written in different ways, they have the same value. Thus, fractions having equal value are called equivalent fractions.

There are two ways of obtaining equivalent fractions.

(a) By multiplying its numerator and denominator by the same number. ·

$$\frac{1}{2} = \frac{1\times2}{2\times2} = \frac{2}{4}, \frac{1}{2} = \frac{1\times3}{2\times3} = \frac{3}{6}, \frac{1}{2} = \frac{1\times4}{2\times4} = \frac{4}{8}$$

$\therefore \frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}$ are all equivalent fractions.

(b) By dividing its numerator and denominator by the same number.

$$\frac{2}{4} = \frac{2\div2}{4\div2} = \frac{1}{2}, \frac{3}{6} = \frac{3\div3}{6\div3} = \frac{1}{2}, \frac{4}{8} = \frac{4\div4}{8\div4} = \frac{1}{2}$$

Ex. Write four equivalent fractions of $\frac{4}{9}$.

Sol. Equivalent fractions of $\frac{4}{9}$ are :

$$\frac{4\times2}{9\times2} = \frac{8}{18}, \frac{4\times3}{9\times3} = \frac{12}{27}, \frac{4\times4}{9\times4} = \frac{16}{36}, \frac{4\times5}{9\times5} = \frac{20}{45}$$

Fraction in its Lowest Term :

A fraction is said to be in its lowest term if the only common factor between the numerator and the denominator of the fraction is 1, i.e., when the numerator and denominator are co-prime. It is also known as the simplest form or the standard form of a fraction.

For example, $\frac{3}{5}$ is a fraction in its lowest term.

A fraction can be reduced to its lowest term by following the steps given below.

1. Find the HCF of the numerator and the denominator.
2. Then divide both the numbers by the HCF.
3. The resultant fraction will be in its lowest term.

Ex. Reduce $\frac{24}{72}$ to its lowest term.

Sol. The HCF of 24 and 72 is 24. Divide both 24 and 72 by 24.

$$\frac{24}{72} = \frac{24 \div 24}{72 \div 24} = \frac{1}{3}$$

LIKE AND UNLIKE FRACTION

Two or more fractions with the same denominators are called like fractions, whereas fractions with different denominators are called unlike fractions.

For example,

$\frac{7}{20}, \frac{13}{20}, \frac{9}{20}, \frac{11}{20}$ are all like fractions as their denominators are all equal.

$\frac{3}{7}, \frac{8}{9}, \frac{3}{8}, \frac{6}{13}$ are all unlike fractions as their denominators are not equal.

Conversion of unlike fractions into like fractions

It is easy to compare, add and subtract like fractions.

So, we must learn to convert unlike fractions to like fractions.

Ex. Convert into like fractions. $\frac{3}{8}, \frac{5}{6}, \frac{7}{12}$

Sol. Find the LCM of the denominators 8,6, and 12.

LCM = 2 × 2 × 2 × 3 = 24

Now the above fractions can be converted into equivalent fractions with 24 as the denominator.

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2 | 8, 6, 12
  2 | 4, 3, 6
    2 | 2, 3, 3
        2, 1, 1
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$$\frac{3}{8} = \frac{3 \times 3}{8 \times 3} = \frac{9}{24}, \frac{5}{6} = \frac{5 \times 4}{6 \times 4} = \frac{20}{24} \text{ and } \frac{7}{12} = \frac{7 \times 2}{12 \times 2} = \frac{14}{24}$$

So, $\frac{9}{24}, \frac{20}{24}$ and $\frac{14}{24}$ are like fractions representing $\frac{3}{8}, \frac{5}{6}$, and $\frac{7}{12}$, respectively.

Ex. Which of the fractions is greater $\frac{3}{8}$ or $\frac{5}{16}$?

Sol. To convert $\frac{3}{8}$ into a fraction with denominator 16, it is converted into an equivalent fraction with denominator 16.

$$\frac{3}{8} = \frac{3 \times 2}{8 \times 2} = \frac{6}{16}$$

So, $\frac{3}{8}$ and $\frac{5}{16}$ will become $\frac{6}{16}$ and $\frac{5}{16}$ respectively.

$\frac{6}{16}$ is greater. hence, $\frac{3}{8}$ is greater than $\frac{5}{16}$

Ex. Arrange in ascending order $\frac{5}{8}, \frac{5}{6}, \frac{7}{4}, \frac{3}{5}$?

Sol. LCM of 8, 6, 4, 5

LCM = 2 X 2 X 2 X 3 X 5 = 120.

$$\frac{5}{8} = \frac{5 \times 15}{8 \times 15} = \frac{75}{120} \ (120 \div 8 = 15)$$

$$\frac{5}{6} = \frac{5 \times 20}{6 \times 20} = \frac{100}{120} \ (120 \div 6 = 20)$$

$$\frac{7}{4} = \frac{7 \times 30}{4 \times 30} = \frac{210}{120} \ (120 \div 4 = 30)$$

$$\frac{3}{5} = \frac{3 \times 24}{5 \times 24} = \frac{72}{120} \ (120 \div 5 = 24)$$

Since 72 < 75 < 100 < 210

$$\frac{72}{120} < \frac{75}{120} < \frac{100}{120} < \frac{210}{120}$$

$$\therefore \frac{3}{5} < \frac{5}{8} < \frac{5}{6} < \frac{7}{4}$$

Representation of a Proper Fraction on a Number Line :

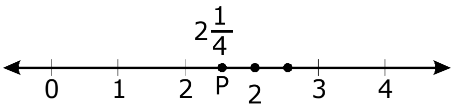
Draw a number line and mark points at equal intervals.

Let these points represent the numbers 0, 1, 2, ... let us find a point on the number line corresponding to the fraction $\frac{2}{7}$. The denominator 7 suggests that the unit distance is to be divided into seven equal parts and 2 of these parts are to be taken. Thus the point P on the following number line represents the fraction $\frac{2}{7}$.

Representation of an Improper Fraction on a Number Line

Let us represent $2\frac{1}{4}$ on a number line.

Draw a number line and mark points at equal intervals.
Let these points represent the numbers 0, 1, 2, 3,



We have learnt that $2\frac{1}{4} = 2 + \frac{1}{4}$ which shows that the number is greater than 2 but less than 3.

So divide the distance between 2 and 3 in 4 equal parts (4 is the denominator of $\frac{1}{4}$). Each part represents $\frac{1}{4}$. The point P in the above figure represents the fraction $2\frac{1}{4}$.

COMPARISON OF FRACTION

(i) Compare two fractions $\frac{3}{7}$ and $\frac{5}{7}$

Since 5 > 3

$\therefore \frac{5}{7} > \frac{3}{7}$

If the denominators of two given fractions are alike, the fraction with greater numerator is greater than the fraction with smaller numerator.

(ii) Compare two fractions $\frac{3}{5}$ and $\frac{3}{9}$.

since 5 < 9

Clearly, $\frac{3}{5} > \frac{3}{9}$

If two different fractions with same numerators and unlike denominators are given, the fraction with smaller denominator is greater than the fraction with greater denominator.

(iii) Compare two fractions $\frac{7}{9}$ and $\frac{5}{12}$.

first we change $\frac{7}{9}$ and $\frac{5}{12}$ to equivalent fractions having like denominators.

the L.C.M. of 9 and 12 = 36

$\frac{7}{9} = \frac{7 \times 4}{9 \times 4} = \frac{28}{36}$ and $\frac{5}{12} = \frac{5 \times 3}{12 \times 3} = \frac{15}{36}$

compare $\frac{28}{36}$ and $\frac{15}{36}$

28 > 15

$\therefore \frac{28}{36} > \frac{15}{36}$ or $\frac{7}{9} > \frac{5}{12}$

OPERATION OF FRACTION

(i) Addition of Fractions with like denominators :

To add two or more fractions having the same denominators, we add the numerators of the given fractions. The number thus obtained become the numerator of the required fraction and the denominator of this fraction is the common denominator of the given fractions.

Ex. Add together : $\frac{4}{18}$ and $\frac{7}{18}$.

Sol. $\frac{4}{18} + \frac{7}{18} = \frac{4+7}{18} = \frac{11}{18}$.

(ii) Addition of Fractions with unlike denominators :

First change all the fractions to their equivalent fractions with common denominator which is the L.C.M. of the denominators of the given fractions and then add them as explained earlier.

Ex.8 Add : $\frac{2}{7}$ and $\frac{3}{14}$ add express the sum in its lowest terms.

Sol. We find the prime factors of 7 and 14.

$7 = 7$

$14 = 7 \times 2$

L.C.M. of 7 and 14 = 7 × 2 = 14

Now $\frac{2}{7} = \frac{2 \times 2}{7 \times 2} = \frac{4}{14}$

$\therefore \frac{2}{7} + \frac{3}{14} = \frac{4}{14} + \frac{3}{14} = \frac{4+3}{14} = \frac{7}{14}$

$\frac{7}{14} = \frac{7 \div 7}{14 \div 7} = \frac{1}{2}$

(iii) Addition of Mixed Numbers

To add two or more mixed numbers, we change them to improper fractions and then add.

Ex.9 Add : $1\frac{1}{8}$ and $3\frac{3}{8}$.

Sol. $1\frac{1}{8} + 3\frac{3}{8} = \frac{9}{8} + \frac{27}{8}$

$= \frac{9+27}{8} = \frac{36}{8}$

$= 4\frac{4}{8} = 4\frac{1}{2}$.

(iv) Subtraction of fractions with like denominators

To subtract a fraction from another fraction of like denominator, we subtract the smaller numerator from the greater numerator. The number thus obtained is the numerator of the required fraction and the denominator of this fraction is the common denominator of the given fractions.

Ex.10 Solve : (a) $\frac{6}{11} - \frac{2}{11}$

Sol. (a) $\frac{6}{11} - \frac{2}{11} = \frac{6-2}{11} = \frac{4}{11}$.

(v) Subtraction of fractions with unlike denominators

To subtract a fraction from another fraction of unlike denominators, we change both the fractions to equivalent fractions of common denominator and then subtract as explained earlier for fractions of like denominators.

Ex. Solve : $\frac{17}{18} - \frac{5}{12}$.

Sol. L.C.M. of 18 and 12 = 2 × 2 × 3 × 3 = 36

Now $\frac{17}{18} = \frac{17 \times 2}{18 \times 2} = \frac{34}{36}$

$\frac{5}{12} = \frac{5 \times 3}{12 \times 3} = \frac{15}{36}$

$\frac{17}{18} - \frac{5}{12} = \frac{34}{36} - \frac{15}{36}$

$= \frac{34-15}{36} = \frac{19}{36}$.

(vi) Subtraction of Mixed Numbers

To subtract mixed numbers, we change them to improper fractions and then subtract.

Ex. Solve : $6\frac{1}{3} - 3\frac{2}{9}$.

Sol. L.C.M. of 3 and 9 = 9.

$6\frac{1}{3} - 3\frac{2}{9} = \frac{19}{3} - \frac{29}{9}$

$= \frac{19 \times 3}{3 \times 3} - \frac{29}{9}$

$= \frac{57}{9} - \frac{29}{9}$

$= \frac{57-29}{9} = \frac{28}{9} = 3\frac{1}{9}$.