OSCILLATIONS

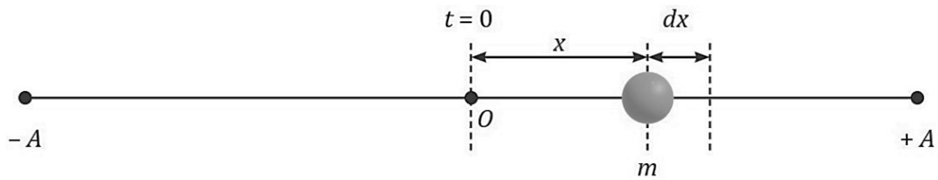
ENERGY IN SIMPLE HARMONIC MOTION

ENERGY OF SHM

POTENTIAL ENERGY

It is known that the negative of work done by the conservative force is given as,

$\Delta U = W_i - W_f$



Imagine a particle undergoing Simple Harmonic Motion (SHM) due to a force represented as $F = -kx$ Let's assume that U_0 and U represent the potential energies of the particle when it's at displacements x_0 and x, respectively. Following the work-energy theorem, we find that:

$U - U_0 = -\int F\,dx = -\int_{x_0}^{x} (-kx)\,dx$
$U - U_0 = \frac{1}{2}kx^2 - \frac{1}{2}k{x_0}^2$

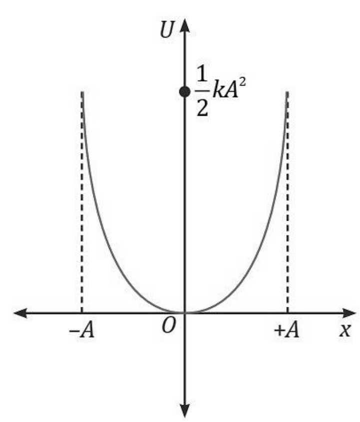
Now, if we take the reference point to be the middle position, where $x_0 = 0$ and $U_0 = 0$,

The potential energy of the particle at any position x is determined by the following formula:

$U(x) = \frac{1}{2}kx^2$

Therefore, the potential energy is the minimum at the mean position and is often taken as zero. It is the maximum at the extremes, i.e., $\frac{1}{2}kA^2$.

In the event that the displacement is delineated as $x = A\,sin\,(\omega t + \phi)$, with the presumption that the midpoint signifies a state of zero potential, the formulation of the potential energy takes the following form:



$U(x) = \frac{1}{2}kA^2 \sin^2(\omega t + \emptyset)$

Which can be further simplified as follows:

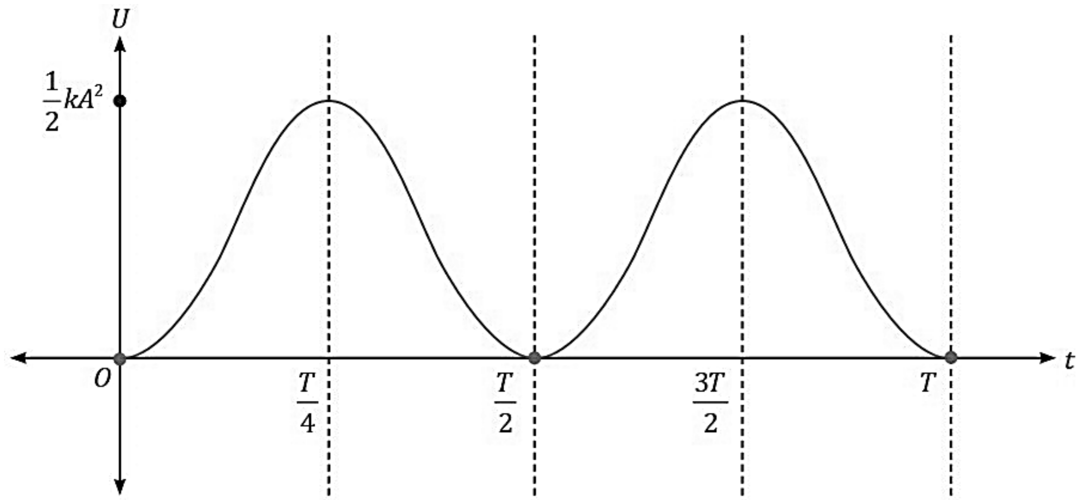
$U(x) = \frac{1}{4}kA^2\big(2\sin^2(\omega t + \emptyset)\big)$

$U(x) = \frac{1}{4}kA^2\big(1 - \cos(2\omega t + 2\emptyset)\big)$

$U(x)\frac{1}{4}kA^2 - \frac{1}{4}kA^2\cos\,(2\omega t + 2\emptyset)$

This reveals that the potential energy behaves like a simple harmonic function as well. The angular frequency of this potential energy's SHM, denoted as U(x), 2ω. Consequently, the frequency of U(x) also doubles, becoming 2f.

The time variation of the potential energy graph is shown in the given figure.



Kinetic energy

We are acquainted with the definition of kinetic energy for a particle with mass m that is in motion with a velocity v, which is stated as follows:

$KE = \frac{1}{2}mv^2$

In the event that the displacement of a particle engaged in executing SHM is elucidated as $x = A\,sin(\omega t + \phi)$, the association between the velocity and the particle's displacement is articulated as follows:

$v = \omega\sqrt{A^2 - x^2}$

The expression for the kinetic energy of the particle executing the SHM is as follows:

$KE = \frac{1}{2}mv^2$

$KE = \frac{1}{2}m\omega^2(A^2 - x^2)$

Since ω^2 was defined as $\frac{k}{m}$ (where k is the constant of SHM),

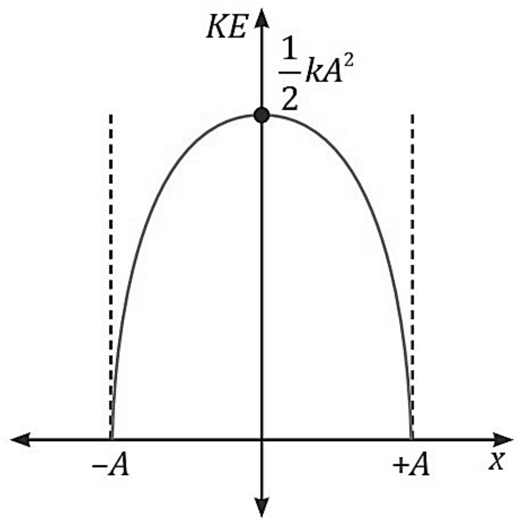
The expression of the kinetic energy can be written as follows:

$KE = \frac{1}{2}k(A^2 - x^2)$

It can be deduced that,

- The highest point of kinetic energy happens at the middle position (x = 0), specifically, $KE_{max} = \frac{1}{2}kA^2$.

- The lowest values of kinetic energy are located at the farthest points ($x = \pm A$), i.e., $KE_{min} = 0$



The velocity of the particle corresponding to the standard equation of the SHM is given by,

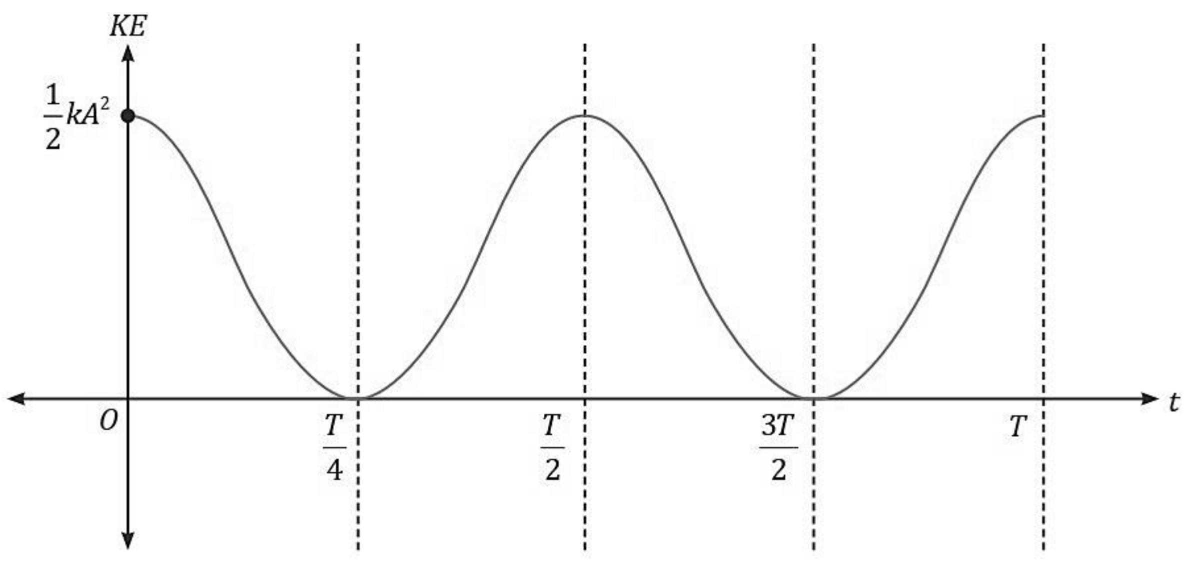
$$v = \frac{dx}{dt} = A\omega\cos(\omega t + \emptyset)$$

Thus, the kinetic energy of the particle as a function of time can be written as follows:

$$KE = \frac{1}{2}mA^2\omega^2\cos^2(\omega t + \emptyset)$$

$$KE = \frac{1}{2}kA^2\cos^2(\omega t + \emptyset)$$

Just like potential energy, the kinetic energy function has a frequency that is twice as fast as the SHM.



Total energy of SHM

The overall mechanical energy of the particle in SHM = potential energy + kinetic energy.

Thus, the total mechanical energy

$E = K + U$

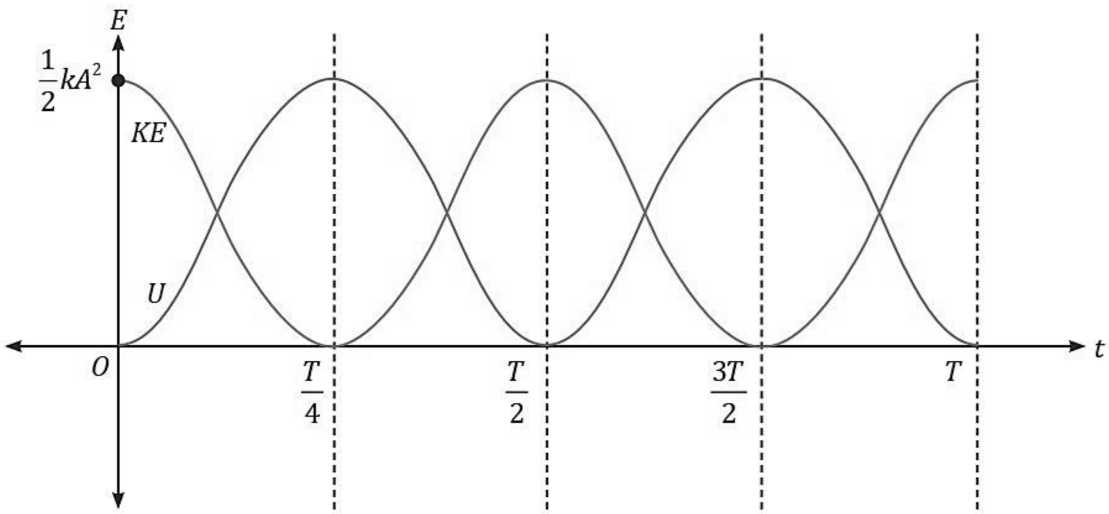
$E = \frac{1}{2}k(A^2 - x^2) + \frac{1}{2}kx^2$

$E = \frac{1}{2}kA^2$

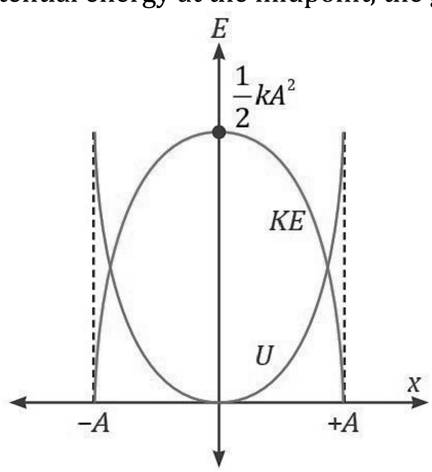
Since both K and A are constants, the total mechanical energy of the particle executing the SHM is constant.

Graphical representation:

The provided figure displays a combined graph showing potential energy, kinetic energy, and total mechanical energy.



If the particle starts with zero potential energy at the midpoint, the graph of E vs x will look like this:



If the particle has some amount of potential energy at the midpoint, the graph of E vs x will appear like this: