WAVES

DISPLACEMENT RELATION IN A PROGRESSIVE WAVE

PROGRESSIVE WAVE

A wave that moves steadily through something without getting smaller or changing its shape is called a traveling wave or a progressive wave. In this part, we will create a formula that tells us how a wave moves through something and what it looks like at any moment as it moves.

Now, let's think about a wave on a string as an example. We'll explain how the movement of any part of the string is connected to time, and we'll also look at how the different parts of the string move all at once at a specific moment in time.

If we call y(x,t) the movement of a spot at a certain place (x) and moment (t) around the up-and-down line (y-axis), and we assume the wave repeats in a regular and wavy pattern like a sine wave, we can describe the movement of that spot at x and t like this:

$$y(x, t) = a \sin (kx - \omega t + \varphi) \text{ ....(a)}$$

We can write the above equation as a linear combination of sine and cosine functions as,

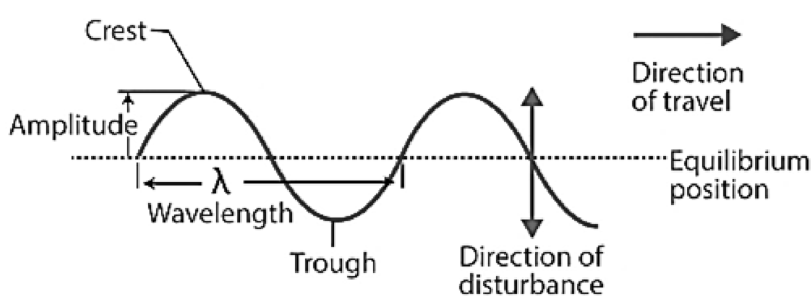
$$y(x, t) = A\sin(kx - \omega t) + B\cos(kx - \omega t) \text{ ...... (b)}$$

$$a = \sqrt{A^2 + B^2} \text{ and } \phi = \tan^{-1}\frac{B}{A}$$

The equations (a) and (b) show a wave that moves sideways along the X-axis. In these equations, y(x, t) tells us how far the parts of the string move at a certain position (x) and any moment (t). This helps us figure out what the wave looks like at any moment.

$$y (x, t) = a \sin (kx + \omega t + \varphi),$$

The equation above shows a wave that's going sideways in the direction of the negative X-axis.



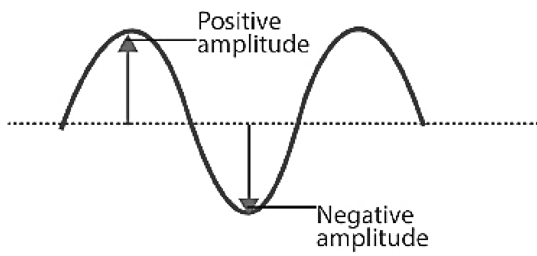
The parameters that describe a harmonic wave entirely are ‘a’, ‘φ’, ‘k’, and ‘ω’, where a is the amplitude, φ is the initial phase change, k is the angular wavenumber, and ω is the angular frequency. Let us now learn in detail what these quantities represent.

Consider the sinusoidal graph shown above. Here, the plot shows a wave travelling in the positive X direction.

The equation above shows a wave that's going sideways in the direction of the negative X-axis.

Amplitude

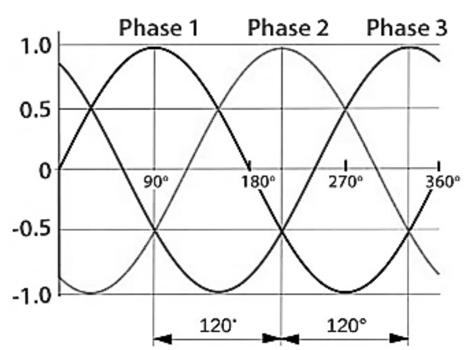
Amplitude is the magnitude of maximum displacement of a particle in a wave from the equilibrium position.



The picture above displays both high and low points in a wavy line, which is how a sine wave looks. But when we talk about how big the up-and-down movement is, we always use a positive number called the amplitude.

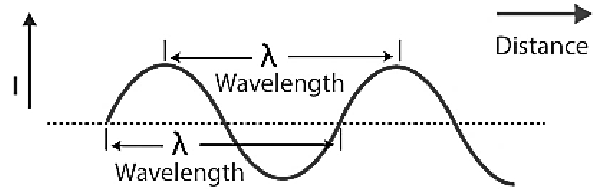
Phase

The part (kx - ωt + φ) in the wavy term sin (kx - ωt + φ) is called the "phase" of the function. It tells us how the wave is moving. When points on a wave move together in the same way, going up and down at the same time, we say they are "in phase" with each other. But when points on a wave move in opposite ways, like one going up while the other goes down, we say they are "in anti-phase" with each other.



Wavelength

Wavelength (λ) is like the space between two of the same parts of a wave, like two high points or two low points, in the direction the wave is moving. It's also how far the wave pattern happens over and over again. We measure it in meters (m).



Wave

Angular wave number

The wavenumber is the spatial frequency of a wave in terms of cycles per unit distance. It can also be defined as the number of waves that exist over a specified distance, analogous to the concept of frequency.

Angular frequency

Angular frequency means how quickly the wave's phase changes in a certain amount of time.

Mathematically we can represent it as,

$$\omega = \frac{2\pi}{T} = 2\pi f$$

Here, T stands for the time it takes for the wave to repeat its pattern, and f is the number of times it repeats in a second, which is the frequency.