DIFFERENTIAL EQUATIONS

GENERAL AND PARTICULAR SOLUTIONS OF A DIFFERENTIAL EQUATION

SOLUTION OF A DIFFERENTIAL EQUATION

Solving or integrating a differential equation involves determining the dependent variable in terms of the independent variable. The solution or integral of a differential equation is a relationship between the dependent and independent variables that is free from derivatives and satisfies the given differential equation.

Therefore, obtaining the solution of $\frac{dy}{dx} = e^x$ involves integrating both sides.

i.e., $y = e^x + c$ and that of, $\frac{dy}{dx} = px + q$ is $y = \frac{px^2}{2} + qx + c$, where c is arbitrary constant.

The primitive of the differential equation is also referred to as its solution, as the differential equation can be seen as a relation derived from it.

There can be three types of solution of a differential equation:

(i) General solution (or complete integral or complete primitive)

A relation in x and y that satisfies a given differential equation and involves exactly the same number of arbitrary constants as the order of the differential equation.

For example, a general solution of the differential equation $\frac{d^2x}{dt^2} = -4x$ is

$x = A \cos 2t + B \sin 2t$ where A and B are the arbitrary constants.

(ii) Particular solution or particular integral

Is the solution to the differential equation derived by substituting specific values for the arbitrary constant in the general solution?

For example, A specific solution to the differential equation is given by $x = 10 \cos 2t + 5 \sin 2t$.

$$\frac{d^2x}{dt^2} = -4x.$$

The general solution of a differential equation can be represented in various forms that are equivalent to each other.

For example $\log x - \log(y+2) = k$ ....(i)

The general solution of the differential equation $xy' = y + 2$, where k is an arbitrary constant, is expressed as equation (i). This solution, as given in equation (i), can also be reformulated as follows:

$$\log\left(\frac{x}{y+2}\right) = k \text{ or } \frac{x}{y+2} = e^k = c_1 \quad .....(ii)$$

Or $x = c_1\ (y+2)$ .....(iii)

Where $c_1 = e^k$ is an additional arbitrary constant, the solution (iii) can also be expressed as:

$$y + 2 = c_2 x$$

Where $c_2 = 1/c_1$ is another arbitrary constant.

Every differential equation we encounter possesses either a unique solution or a family of solutions. For instance, the differential equation $\left|\frac{dy}{dx}\right| + |y| =$ has solely the trivial solution, wherein y equals 0.

The differential equation $\left|\frac{dy}{dx}\right| + |y| + c = 0, c > 0$ has no solution.

(iii) Singular Solution

It cannot be derived from the general solution. Geometrically, the general solution serves as the envelope for the singular solution.