# DIFFERENTIAL EQUATIONS

## FORMATION OF A DIFFERENTIAL EQUATION WHOSE GENERAL SOLUTION IS GIVEN

### FORMATION OF A DIFFERENTIAL EQUATION

To derive a differential equation with a solution that

$$f(x_1, y_1, c_1, c_2, c_3........., c_n) = 0$$

Involving $c_1$, $c_2$, $c_n$, which are 'n' arbitrary constants, we need to eliminate these 'n' constants, and for this purpose, we require (n+1) equations.

A differential equation is derived in the following manner:

(A) Take the given equation and differentiate it with respect to the independent variable (let's say x) as many times as there are independent arbitrary constants in the equation.

(B) Remove the arbitrary constants.

(C) The resulting eliminate represents the desired differential equation.

A differential equation depicts a collection of curves, each adhering to shared properties. This can be viewed as the geometric interpretation of the differential equation.

For n differentiations, the resulting equation must include a derivative of nth order, i.e., equal to the number of independent arbitrary constants.

**Ex.1** Derive a differential equation for the family of straight lines that pass through the origin.

**Sol.** The family of straight lines passing through the origin is represented by the equation y = mx, where 'm' is a parameter. Upon differentiation with respect to x,

$$\frac{dy}{dx} = m$$

By removing 'm' from both equations, we acquire

$$\frac{dy}{dx} = \frac{y}{x}$$

Which is the required differential equation.

**Ex.2** Determine the differential equation for all parabolas with axes parallel to the x-axis and a given latus rectum of length 'a'.

**Sol.** The equation of a parabola with an axis parallel to the x-axis and a latus rectum of length 'a' is $(y - \beta)^2 = a\,(x - a)$

Upon differentiation of both sides, we obtain:

$$2\left(y-\beta\right)\frac{dy}{dx}=a$$

Upon further differentiation,

$$2\left(y-\beta\right)\frac{d^2y}{dx^2}+2\left(\frac{dy}{dx}\right)^2=0$$

$$a\frac{d^2y}{dx^2}+2\left(\frac{dy}{dx}\right)^3=0$$

**Ex.3** Determine the differential equation whose solution represents the family:

$y = a\cos qx + b\sin qx$, where $q$ = fixed constant

**Sol.** $y = a\cos qx + b\sin qx$,

$q$ = fixed constant .... (i)

Differentiating,

We get $$\frac{dy}{dx}=-qa\sin qx+qb\cos qx$$

Again differentiating,

We get $$\frac{d^2y}{dx^2}=-q^2a\cos qx-q^2b\sin qx$$

Using equation (i),

We get $$\frac{d^2y}{dx^2}=-q^2y$$