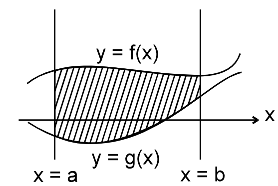
APPLICATION OF INTEGRALS

AREA BETWEEN TWO CURVES

Area between two curves

If $f(x) \geq g(x)$ for $x \in [a,b]$ then the area enclosed by the curves (graphs) $y = f(x)$ and $y = g(x)$ between the ordinates $x = a$ and $x = b$ is given by:



$$x = b \text{ is } \int_a^b \left(f(x) - g(x)\right)dx$$

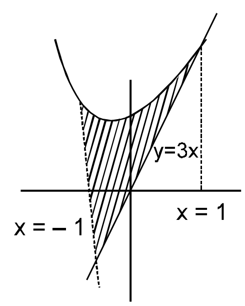
Ex.1 Determine the area enclosed by the curve (graph) $y = x^2 + x + 1$ and its tangent at (1,3) between the ordinates $x = -1$ and $x = 1$.

Sol.

$$\frac{dy}{dx} = 2x + 1$$

$$x = 1$$

$$\frac{dy}{dx} = 3$$



Equation of tangent is

$$y - 3 = 3(x - 1)$$

$$y = 3x$$

Required area $= \displaystyle\int_{-1}^{1} \left(x^2 + x + 1 - 3x\right)dx$

$$= \int_{-1}^{1} \left(x^2 - 2x + 1\right)dx$$

$$= \left.\frac{x^3}{3} - x^2 + x\right]_{-1}^{1}$$

$$= \left(\frac{1}{3} - 1 + 1\right) - \left(-\frac{1}{3} - 1 - 1\right)$$

$$= \frac{2}{3} + 2 = \frac{8}{3}$$

Note : The area enclosed between the curves y = f(x) and y = g(x) between the ordinates (x = a) and (x = b) is

$$\int_a^b |f(x)-g(x)|\,dx\ .$$

Ex.2 Find the area of the region enclosed by the curves y = sin x, y = cos x, and the ordinates $x = 0, x = \frac{\pi}{2}$

Sol.

$$\int_0^{\frac{\pi}{2}} \left|\sin x-\cos x\right| dx$$

$$\int_0^{\frac{\pi}{4}} \left(\cos x-\sin x\right) dx+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left(\sin c-\cos x\right) dx$$

$$=2\left(\sqrt{2}-1\right)$$