INTEGRALS

INTEGRATION BY PARTS

Integration by parts:

The integration of the product of two functions, f(x) and g(x), can be accomplished using the formula:

$$\int(f(x)g(x))dx = f(x)\int(g(x))dx - \int\left(\frac{d}{dx}(f(x))\int(g(x))dx\right)dx$$

(i) Upon determining the integral $\int g(x)\,dx$, it will be devoid of any arbitrary constants.

(ii) $\int g(x)\,dx$ Should be considered identical in both instances.

(iii) The selection of f(x) and g(x) can be guided by the ILATE rule. The function that appears later is treated as the integral function (g(x)).

I → Inverse function

L → Logarithmic function

A → Algebraic function

T → Trigonometric function

E → Exponential function

Ex.1 Evaluate: $\int x\log_e x\,dx$

Sol. Let $I = \int g(x)\,dx$

$$-\log_e x\int x\,dx - \int\left\{\frac{d}{dx}(\log x)\int x\,dx\right\}dx$$

$$\log_e x\left(\frac{x^2}{2}\right) - \int\frac{1}{x}\times\frac{x^2}{2} - dx$$

$$= \frac{x^2}{2}\log_e x - \frac{x^2}{4} + C$$

Ex.2 Evaluate: $\int x\,\text{In}(1+x)\,dx$

Sol. Let $I = I = \int x\,\text{In}(1+x)\,dx$

$$\text{In}(x+1)\frac{x^2}{2} - \int \frac{1}{x+1}.\frac{x^2}{2}dx$$

$$\frac{x^2}{2}\text{In}(x+1) - \frac{1}{2}\int \frac{x^2}{x+1}dx$$

$$\frac{x^2}{2}\text{In}(x+1) - \frac{1}{2}\int \frac{x^2-1+1}{x+1}dx$$

$$\frac{x^2}{2}\text{In}(x+1) - \frac{1}{2}\int \left(\frac{x^2-1}{x+1} + \frac{1}{x+1}\right)dx$$

$$\frac{x^2}{2}\text{In}(x+1) - \frac{1}{2}\int \left((x-1) + \frac{1}{x+1}\right)dx$$

$$\frac{x^2}{2}\text{In}(x+1) - \frac{1}{2}\int \left(\frac{x^2}{2} - x + \text{In}|x+1|\right) + C$$

Ex.3 Evaluate: $\int e^{2x}\sin 2x\,dx$

Sol. We know that $\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}\ (a\sin bx - b\cos bx) + C$

$$a = 2$$

and $b = 2 = \frac{e^{2x}}{8}\ (2\sin 2x - 2\cos 2x) + C$

Note :

(i) $\int e^x\,[f(x) + f'(x)]\,dx = e^x.\,f(x) + C$

(ii) $\int [f(x) + xf'(x)]\,dx = x\,f(x) + C$

Ex.4 Evaluate: $\int \left[\text{In}\ (\text{In}\,x) + \frac{1}{(\text{In}x)^2}\right]dx$

Sol. Let $I = \int \left(\text{In}(\text{In}x) + \frac{1}{(\text{In}x)^2}\right)dx$

put $x = e^t$

$$dx = e^t dt$$

$$I = \int e^t \left(\text{Int} + \frac{1}{t^2} \right) dt$$

$$\int e^t \left(\text{Int} - \frac{1}{t} + \frac{1}{t} + \frac{1}{t^2} \right) dt$$

$$e^t \left(\text{Int} - \frac{1}{t} \right) + C$$

$$x \left[\text{In}(\text{Inx}) - \frac{1}{\text{Inx}} \right] + C$$