INTEGRALS

METHODS OF INTEGRATION

Integration by substitution:

If we perform the substitution $\phi(x) = t$ in an integral, then

(i) Every occurrence of x will be substituted with the new variable t.

(ii) The differential dx is also expressed in terms of dt.

Ex.1 Evaluate: $\int \frac{\sec^2 x}{3+\tan x} dx$

Sol.

$$I = \int \frac{\sec^2 x}{3+\tan x} dx$$

$$3+\tan x = t$$

$$\sec^2 x dx = dt$$

$$\int \frac{dt}{t} = \text{In}t + C$$

$$\text{In}|(3+\tan x)| + C$$

Ex.2 Determine: $\int \frac{1}{1+e^{-x}} dx$

Sol.

$$I = \int \frac{1}{1+e^{-x}} dx$$

$$\int \frac{e^x}{e^x+1}$$

$$\int \frac{e^x}{e^x+1}$$

$$\int \frac{\frac{d}{dx}\left(e^x+1\right)}{(e^x+1)}$$

$$\log_e\left|e^x+1\right| + C$$

Ex.3 Find: $\int \tan^4 x\,dx$

Sol.
$$\int \tan^4 x dx = \int \tan^2 x \cdot \tan^2 x dx$$
$$\int \tan^2 x\left(\sec^2 x - 1\right)dx$$
$$\int \tan^2 x \sec^2 x dx - \int \tan^2 x dx$$
$$\int \tan^2 x \sec^2 x dx - \int \left(\sec^2 x - 1\right)dx$$
$$\frac{\tan^3 x}{3} - \tan x + x + C$$

Ex.4 Evaluate: $\int \frac{x}{x^4 + x^2 + 1}dx$

Sol.
$$I = \int \frac{x}{x^4 + x^2 + 1}dx$$
$$\int \frac{x}{\left(x^2\right)^2 + x^2 + 1}dx$$
$$x^2 = t$$
$$x \cdot dx = \frac{dt}{2}$$
$$I = \frac{1}{2}\int \frac{1}{t^2 + t + 1}dt$$
$$\frac{1}{2}\int \frac{1}{\left(t + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}dt$$
$$\frac{1}{2} \cdot \frac{1}{\frac{\sqrt{3}}{2}} \tan^{-1}\left(\frac{t + \frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + C$$
$$\frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{2t + 1}{\sqrt{3}}\right) + C$$
$$\frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{2x^2 + 1}{\sqrt{3}}\right) + C$$