PROBABILITY

MULTIPLICATION THEOREM

CONDITIONAL PROBABILITY AND MULTIPLICATION THEOREM

Let A and B be two events with $P(A) > 0$. Then $P(B|A)$ denotes the conditional probability of B given that A has occurred. Since A is known to have occurred, it becomes the new sample space, replacing the original S. This leads to the definition:

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

This expression is called the conditional probability of B given A $P(A \cap B) = P(A)P(B|A)$, also known as compound probability or the multiplication theorem. It states that the probability of both A and B occurring is equal to the probability of A occurring multiplied by the probability of B occurring given that A has occurred.

Note: For any three events A_1, A_2, A_3

We have $$P(A_1 \cap A_2 \cap A_3) = P(A_1)P(A_2 | A_1)P(A_3 | (A_1 \cap A_2))$$

Ex.1 If $p\left(\frac{A}{B}\right) = 0.2$ and $P(B) = 0.5$ and $P(A) = 0.2$. Find $P(A \cap B)$.

Sol. $$P(A \cap \overline{B}) = P(A) - P(A \cap B)$$

Also $$p\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cap B) = 0.1$$

From given data, $$P(A \cap \overline{B}) = 0.1$$

Ex.2 When two dice are thrown, determine the probability of obtaining a sum of 8, given that the second die always shows a 4.

Sol. $$n\ (A) = \{(1,4),(2,4),(3,4),(4,4),(5,4),(6,4)\}$$

$$n\ (A) = 6$$

and B be the event of occurrence of such numbers on both dice whose sum is

$$8 = \{(6,2),(5,3),(4,4),(3,5),(2,6)\}.$$

Thus, $$A \cap B = \{(4,4)\}$$

$$n(A \cap B) = 1$$

$$p\left(\frac{B}{A}\right) = \frac{n(A \cap B)}{n(A)} = \frac{1}{6}$$

$$\frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{36}}{\frac{6}{36}} = \frac{1}{6}$$

Ex.3 In a bag containing 3 red, 6 white, and 7 blue balls, two balls are drawn successively without replacement. Determine the probability of drawing the first ball as white and the second ball as blue.

Sol. Let A represent the event of drawing the first ball as white, and let B denote the event of drawing the second ball as blue. In this scenario, events A and B are dependent.

$$P(A) = \frac{6}{16}, P(B|\ A) = \frac{7}{15}$$

$$P(AB) = P(A) \cdot P(B|\ A)$$

$$= \frac{6}{16} \times \frac{7}{15} = \frac{7}{40}$$