THREE DIMENSIONAL GEOMETRY

EQUATION OF A LINE IN SPACE

STRAIGHT LINE
Definition

A straight line in space is defined by the intersection of two non-parallel planes. Consequently, the equation of a straight line serves as a solution to the system formed by the equations of these two planes:

$$a_1x + b_1y + c_1z + d_1 = 0;$$
$$a_2 x + b_2y + c_2z + d_2 = 0$$

This form is also referred to as the unsymmetrical form.

Some Particular Straight Lines:

	Straight Lines	Equation
1.	Through The Origin	$y = mx, z = nx$
2.	X-Axis	$y = 0, z = 0$ or $\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$
3.	Y-Axis	$x = 0, z = 0$ or $\frac{x}{0} = \frac{y}{1} = \frac{z}{0}$
4.	Z-Axis	$x = 0, y = 0$ or $\frac{x}{0} = \frac{y}{0} = \frac{z}{1}$
5.	Parallel To X-Axis	$y = p, z = q$
6.	Parallel To Y-Axis	$x = h, z = q$
7.	Parallel To Z-Axis	$x = h, y = p$

Equation of A Line

1. **A line in space** is defined by the intersection of two non-parallel planes. Thus, the equation of a straight line serves as a solution to the system formed by the equations of these two planes.

$$a_1x + b_1y + c_1z + d_1 = 0$$
$$a_2x + b_2y + c_2z + d_2 = 0.$$

This form is also referred to as the non-symmetrical form.

2. **Point-slope form**: If a point (x_1, y_1, z_1) lies on the line, and ℓ, m, n are the direction cosines of the line, then its equation is given by...

$$\frac{x-x_1}{\ell}=\frac{y-y_1}{m}=\frac{z-z_1}{n}=r \qquad \text{(Say)}$$

It should be noted that p($x_1 + \ell r$, $y_1 + mr$, $z_1 + nr$) is a general point on this line at a distance r from the point A (x_1, y_1, z_1) i.e. AP = r. one should note that for AP = r; ℓ , m, n must be D.C.'s not d.r.'s. If a, b, c are direction ratios of the line, then equation of the line is

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}=r \qquad \text{But here } AP \neq r$$

3. **Vector equation:** Vector representation: the vector equation of a straight line passing through a fixed point with the position vector $\vec{a}$ and parallel to a given vector $\vec{b}$ is...

$$\vec{r}=\vec{a}+\lambda\vec{b}$$

Where λ is a scalar.

4. The equation of the line that passes through the points and (x_1, y_1, z_1) is.

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

5. Vector equation of a straight line passing through two points with position vectors $\vec{a}\,\&\,\vec{b}$ is...

$$\vec{r}=\vec{a}+\lambda(\vec{b}-\vec{a})$$

6. Reduction of Cartesian form of equation of a line to vector form & vice versa.

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

$$\vec{r}=(x_1\hat{i}+y_1\hat{j}+z\hat{k})+\lambda(a\hat{i}+b\hat{j}+c\hat{k})$$

	STRAIGHT LINE	EQUATION
1.	through origin	y = mx and z = nx
2.	X – axis	y = 0 and z = 0
3.	Y – axis	x = 0 and z = 0
4.	Z – axis	x = 0 and y = 0
5.	parallel to X – axis	y = p, z = q
6.	parallel to Y – axis	x = h, z = q
7.	parallel to Z – axis	x = h, y = p

Ex.1 Determine the vector form and Cartesian form of the equation for the line passing through the points (3, 4, 7) and (1,–1, 6).

Sol. Let $A \equiv (3, 4, -7),\ B \equiv (1, -1, 6)$

Now $\vec{a} = \overrightarrow{OA} = 3\vec{i} + 4\vec{j} - 7\vec{K}$

$$\vec{b} = \overrightarrow{OB} = \vec{i} - \vec{j} + 6\vec{K}$$

Equation of the line through A ($\vec{a}$) and B ($\vec{b}$) is $\vec{r} = \vec{a} + t(\vec{b} - \vec{a})$

$$\vec{r} = 3\vec{i} + 4\vec{j} - 7\vec{k} + t(-2\vec{i} - 5\vec{j} + 13\vec{k}) \qquad \dots (1)$$

Equation in Cartesian form:

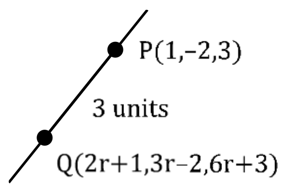
Equation of AB is

$$\frac{x-3}{3-1} = \frac{y-4}{4+1} = \frac{z+7}{-7-6}$$

$$\frac{x-3}{2} = \frac{y-4}{5} = \frac{z+7}{-13}$$

Ex.2 Determine the coordinates of points on the line that are 3 units away from the point (1, –2, and 3).

Sol. Here $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{6}$ ... (i)



Is the given straight line

Let, P = (1,–2,3) on the straight line

Here direction ratios of line (i) are (2, 3, 6)

Direction cosines of line (i) are: $\frac{2}{7}, \frac{3}{7}, \frac{6}{7}$

Equations of line (i) any may be written as

$$\frac{x-1}{\frac{2}{7}} = \frac{y+2}{\frac{3}{7}} = \frac{z-3}{\frac{6}{7}} \qquad \dots\dots (ii)$$

Co-ordinates of any point on the line (ii) may be taken as

$$\left(\frac{2}{7}r + 1, \frac{3}{7}r - 2, \frac{6}{7}r + 3\right)$$

Let, $Q\left(\frac{2}{7}r + 1, \frac{3}{7}r - 2, \frac{6}{7}r + 3\right)$

Given $|\vec{r}| = 3$

$$r = \pm 3$$

Putting the value of r, we have $Q\left(\frac{13}{7}, \frac{5}{7}, \frac{39}{7}\right)$

$$Q = \left(\frac{1}{7}, \frac{23}{7}, \frac{3}{7}\right)$$