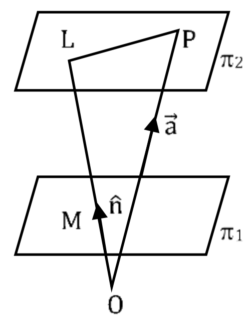
THREE DIMENSIONAL GEOMETRY

DISTANCE OF A POINT FROM A PLANE (FOR COMPETITIVE EXAM)

DISTANCE OF A POINT FROM A PLANE

(a) **To determine the distance of a point with position vector $\vec{a}$ from the plane with the equation $\vec{r}.\vec{n} = p$**



Let π_1 be the plane $\vec{r}.\vec{n} = p$ ... (1)

Let P be the point with the position vector. $\vec{a}$.

Consider the plane π_2 through P parallel to the plane π_1.

$\therefore$ The unit normal to π_2 plane is also $\hat{n}$.

$\therefore$ Its equation is $(\vec{r} - \vec{a}) \cdot \hat{n} = 0$

$\vec{r}.\vec{n} - \vec{a}.\vec{n} = 0 \qquad \vec{r}.\vec{n} = \vec{a}.\vec{n}$

Distance OL if this planes from the origin $O = \vec{a}.\vec{n}$

Case I. When P and O are on opposite sides of π_1 the distance LM of P from the plane π_1 is:

$$\vec{r}.\vec{n} = d.$$

Case II. When P and O are on the same side of π_1 the above formula yields a negative result.

(b) **To determine the perpendicular distance of the point x_1, y_1, z_1 from the plane $lx = my + nz = p$ where $p > 0$.**

Let P (x_1, y_1, z_1) be the given point.

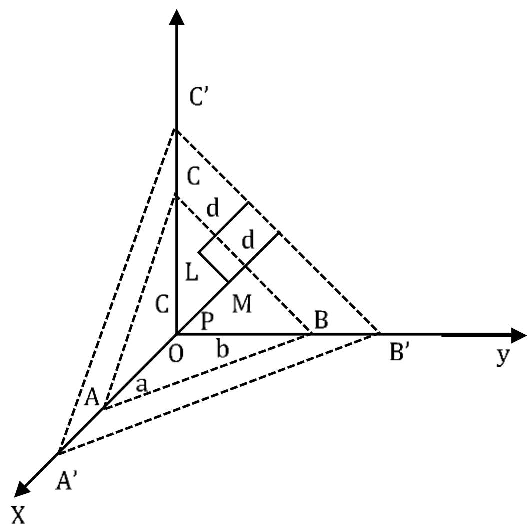
Consider the plane $lx = my = nz = p$ $(p > 0)$, denoted as plane ABC.

Thro, P, draw PL $\perp$ on the plane ABC. Let $= |\overrightarrow{PL}| = d$ (say).

Let OM be Perp. From O on the plane ABC so that $|\overrightarrow{OM}| = p$.

Thus the direction-cosines of $\overrightarrow{OM}$ are $<l, m, n>$

Through point P, draw a plane A'B'C' parallel to the given plane ABC in a way that it intersects the extension of line OM at point N.



Thus $\left|\overrightarrow{ON}\right| = \left|\overrightarrow{OM}\right| + \left|\overrightarrow{MN}\right| = \left|\overrightarrow{OM}\right| + \left|\overrightarrow{LP}\right| = p + d$

The equation of the plane A'B'C' is lx + my + nz p + d.

Since this plane passes thro, P (x_1, y_1, z_1)

$\therefore \quad lx_1 + my_1 + nz_1 = p + d.$

Therefore, $d = lx_1 + my_1 + nz_1 - p$, the numerical value of which provides the desired perpendicular distance.

Rule To Write Down

Substitute the coordinates of the point into the left-hand side of the equation of the plane (after setting the right-hand side equal to 0).

The outcome is the sought-after perpendicular distance.

(C) To determine the perpendicular distance of the point (x_1, y_1, z_1) from the plane $ax + by + cz + d = 0$

The given equation of the plane is $ax + by + cz + d = 0$ ... (1)

To reduce (1) to normal form:

Dividing (1) by $\sqrt{a^2+b^2+c^2}$ we get:

$$\frac{a}{\sqrt{a^2+b^2+c^2}}x+\frac{b}{\sqrt{a^2+b^2+c^2}}y+\frac{c}{\sqrt{a^2+b^2+c^2}}z+\frac{d}{\sqrt{a^2+b^2+c^2}}=0 \quad \text{... (2)}$$

(2) is of the normal form if we transpose the constant terms to RHS.

If D is the required perpendicular distance, then

$$D=\frac{a}{\sqrt{a^2+b^2+c^2}}x_1+\frac{b}{\sqrt{a^2+b^2+c^2}}y_1+\frac{c}{\sqrt{a^2+b^2+c^2}}z_1+\frac{d}{\sqrt{a^2+b^2+c^2}}.$$

$$=\pm\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}$$ [Consider both sigs of d]

Hence, $$D=\frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$$

Note:

Step.1. Set the right-hand side (RHS) of the equation of the plane to zero.

Step.2. Substitute the coordinates of the point into the left-hand side (LHS) of the equation of the plane.

Step.3. Divide the result by $\sqrt{(\text{ coeff.of x})^2+(\text{ coeff.of y})^2+(\text{ coeff.of z})^2}$.

The numerical value of the outcome represents the desired perpendicular distance.