THREE DIMENSIONAL GEOMETRY

COPLANARITY OF TWO LINE

CO- PLANARITY OF TWO LINES

Consider the two lines:

$$\vec{r} = \overrightarrow{a_1} + \lambda \overrightarrow{b_1} \qquad \ldots (1)$$

$$\vec{r} = \overrightarrow{a_2} + \mu \overrightarrow{b_2} \qquad \ldots (2)$$

1. Passes through point A with the position vector $\overrightarrow{a_1}$ and is parallel to vector $\overrightarrow{b_1}$ and
2. Passes through point B with the position vector $\overrightarrow{a_2}$ and is parallel to vector to $\overrightarrow{b_2}$.

Then $\overrightarrow{AB} =$ p.v. of B $-$ p.v. of A $= \overrightarrow{a_2} - \overrightarrow{a_1}$.

The given lines are coplanar if and only if $\overrightarrow{AB}$ is prep to $\vec{b}_1 \times \vec{b}_2$

i.e. $$\overrightarrow{AB} \cdot \left(\overrightarrow{b_1} \times \overrightarrow{b_2} \right) = 0$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1} \right) \cdot \left(\overrightarrow{b_1} \times \overrightarrow{b_2} \right) = 0$$

The shortest distance, denoted as 'd,' between (1) and (2) is zero.

CARTESIAN FORM:

Consider A and B have co-ordinate (x_1, y_1, z_1) and (x_2, y_2, z_2) respectively.

Let $\overrightarrow{b_1}$ and $\overrightarrow{b_2}$ have direction – ratios:

$< a_1, b_1, c_1 >$ And $< a_2, b_2, c_2 >$ respectively.

$$\overrightarrow{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k},$$

$$\overrightarrow{b_1'} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}..$$

$$\overrightarrow{b_2'} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}.$$

The given line are coplanar if and only if $\overrightarrow{AB} \cdot \left(\overrightarrow{b_1} \times \overrightarrow{b_2} \right) = 0$

This can be represented in Cartesian form as:

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} - 0.$$

Ex.1 Demonstrate that the line:

$$\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5} \text{ and } \frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$$

Are situated in the same plane; additionally, determine the equation of the plane.

Sol. Comparing the given equation with.

$$\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$$

$$\frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2},$$

$$d = \left| \frac{\vec{b} \times \left(\overrightarrow{a_2} - \overrightarrow{a_1} \right)}{|\vec{b}|} \right|$$

$$= \frac{-9\hat{i} + 14\hat{j} - 4\hat{k}}{7}$$

$$= \frac{1}{7} |-9\hat{i} + 14\hat{j} - 4\hat{k}|$$

$$= \frac{1}{7} \sqrt{81 + 196 + 16}$$

$$= \frac{1}{7} \sqrt{293} \text{ units.}$$