THREE DIMENSIONAL GEOMETRY

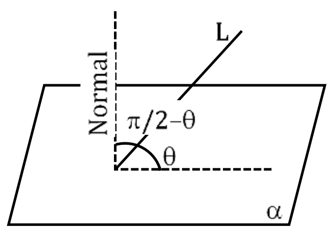
ANGEL BETWEEN A LINE AND A PLANE

(FOR COMPETITIVE EXAM)

ANGEL BETWEEN A LINE AND A PLANE

Let α represent the plane, and L (not parallel to α) denote the line. The angle between L and α is defined as the complement of the acute angle between the normal to the plane α and the line L.

Thus, if $\theta \left(0 < \theta < \frac{\pi}{2}\right)$ is the angle between α and L, then $\left(\frac{\pi}{2} - \theta\right)$ is the acute angle between the normal to the plane α and L.



Note: If L is perpendicular to α then $\theta = \frac{\pi}{2}$

To determine the angle between the line $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ and the plane $ax + by + cz + d = 0$.

The given line is $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$.

The direction ratios of the line are represented by $<l, m, n>$.

The given plane is $ax + by + cz + d = 0$.

Direction-ratios of the normal to the plane are $<a, b, c>$

If 'θ' $\left(0 < \theta < \frac{\pi}{2}\right)$ is the angle between the given line and plane, then $\left(\frac{\pi}{2} - \theta\right)$ is the acute angle between the given line and the normal to the given plane.

$$\cos\left(\frac{\pi}{2} - \theta\right) = \frac{|al + bm + cn|}{\sqrt{l^2 + m^2 + n^2}\sqrt{a^2 + b^2 + c^2}}$$

$$\sin\theta = \frac{|al + bm + cn|}{\sqrt{l^2 + m^2 + n^2}\sqrt{a^2 + b^2 + c^2}}$$

$$\theta = \sin^{-1}\frac{|al+bm+cn|}{\sqrt{l^2+m^2+n^2}\sqrt{a^2+b^2+c^2}}, 0<\theta<\frac{\pi}{2},$$

This is the required angle between the line and the plane.

VECTORIALLY:

Let the given plane be $\vec{r}.\vec{n} = p$ ... (1)

And the given line is parallel to.$\vec{b}$.

If 'ϕ' is the angle between the normal to the plane $\vec{n}$ and the given line (parallel to ($\vec{b}$).

Then $\vec{n}.\vec{b} = nb\cos\phi$.

Where $|\vec{n}| = n$ and $|\vec{n}| - b$.

$$\cos\phi = \frac{\vec{n}\cdot\vec{b}}{nb}$$

$$\phi = \cos^{-1}\left(\frac{\vec{n}\cdot\vec{b}}{nb}\right)$$

If θ is the angle between the line and the plane, then:

$$\theta = \frac{\pi}{2} - \phi = \frac{\pi}{2} - \cos^{-1}\left(\frac{\vec{n}\cdot\vec{b}}{nb}\right) = \sin^{-1}\left(\frac{\vec{n}\cdot\vec{b}}{nb}\right)$$