THREE DIMENSIONAL GEOMETRY

ANGEL BETWEEN TWO PLANES (FOR COMPETITIVE EXAM)

ANGEL BETWEEN TWO PLANES:

Consider a_1 and a_2 as the two provided planes. The angle between these two planes is zero when a_1 and a_2 are parallel. When a_1 and a_2 are not parallel, then:

"The angle between two planes is defined to be the angle between their normal."

Thus, if θ_1 and θ_2 are the two angles between two non-parallel planes, then the relation is: $\theta_1 + \theta_2 = \pi, 0 < \theta_1, \theta_2 < \pi$

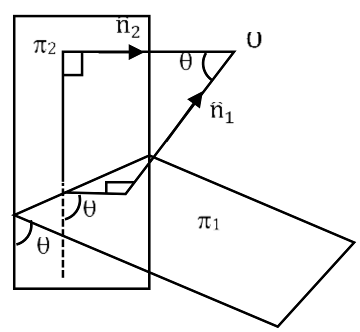
(a) To find the angel between the two planes $\vec{r}.\hat{n}_1 = p_1$ And $\vec{r}.\hat{n}_2 = p_2$

Consider π_1 and π_2 as two given planes.

Then the angle between π_1 and π_2 equals the angle between the normals to planes π_1 and π_2 i.e., the angle between $\hat{n}_1$ and $\hat{n}_2$.

If 'θ' is the angle between planes π_1 and π_2. Then $\hat{n}_1$ and $\hat{n}_2 = \cos\theta$

$$[\therefore \hat{n}_1.\hat{n}_2 = (l)(l)\cos\theta = \cos\theta]$$



Cor.1. When phase are perpendicular.

Here $\theta = \pi/2$. Then $\hat{n}_1.\hat{n}_2 = 0$

Cor.2. When plane are parallel.

Here $\theta = 0°$, Then $\hat{n}_1.\hat{n}_2$

(b) To find the angel between the two planes:

$$a_1x + b_1y + c_1z + d_1 = 0 \text{ and } a_2x + b_2y + c_2z + d_2 = 0.$$

This given plane are $a_1x + b_1y + c_1z + d_1 = 0$ ... (1)

And $a_2x + b_2y + c_2z + d_2 = 0$ ... (2)

The direction ratios of the normal to these planes are $< a_1, b_1, c_1 >$ and $< a_2, b_2, c_2 >$ repectively.

But the angle between the two planes is equal to the angle between their normals.

∴ If 'θ' be the angel between the two planes, Then $$\cos\theta = \pm\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1{}^2+b_1{}^2+c_1{}^2}\sqrt{a_2{}^2+b_2{}^2+c_2{}^2}}.$$

Note: If the two planes are not perpendicular, then the acute angle between them is given by:

$$\cos\theta = \frac{|a_1a_2+b_1b_2+c_1c_2|}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

And the obtuse angle between them is given by:

$$\cos\theta = -\frac{|a_1a_2+b_1b_2+c_1c_2|}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

Cor.1. Condition of Parallelism.

The two planes are parallel if and only if the normal to these planes are parallel, if and only if: $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

Cor.2. Condition of Perpendicularity.

The two planes are perpendicular iff $\theta_1=\theta_2=\frac{\pi}{2}$ iff $\cos\theta_1=\cos\theta_2=0$

Iff $a_1a_2+b_1b_2+c_1c_2=0$.

Note: Two planes $a_1x+b_1y+c_1z+d_1=0$ and $a_2x+b_2y+c_2z+d_2=0$ are identical iff

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}=\frac{d_1}{d_2}$$

Cor.3. Equation of any plane parallel to a given plane.

The equation of any plane parallel to the plane $ax+by+cz+d=0$ is as

$ax+by+cz+k=0$, where K is any constant.

For, the planes $ax+by+cz+d=0$, $ax+by+cz+k=0$, are parallel because

$\frac{a}{a}=\frac{b}{b}=\frac{c}{c}=1$ Which is true.

Rule of write:

In the equation of the given plane, only the constant term is altered to a new constant.