VECTOR ALGEBRA

TYPES OF VECTORS

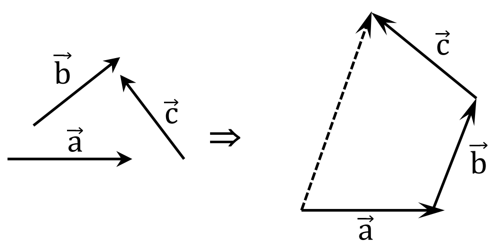
CLASSIFICIATION OF VECTORS

Equal vectors:

Two vectors are considered equal when they share identical magnitude, direction, and represent the same physical quantity.

Free vectors:

A vector is termed a free vector if it can be shifted to any location in space without altering its magnitude or direction. Essentially, the initial point of a free vector can be positioned anywhere in space while maintaining its original magnitude and direction.

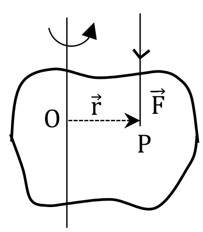


Localised vectors: (FOR COMPETITIVE EXAM)

A vector is categorized as a localized vector if, for a specified magnitude and direction, its initial point is fixed in space.

Examples : Torque, Moment of Inertia etc.

Unless & until stated, vectors are treated as free vectors.



Zero vector

A vector with zero magnitude, meaning that it has identical initial and terminal points, is referred to as a zero vector, denoted by O. The direction of the zero vector is undetermined.

Unit Vector

A vector with a magnitude of one in the direction of another vector is termed a unit vector along that direction, and it is symbolically represented by the symbol $\hat{c}$.

$$\hat{a} = \frac{\vec{a}}{|a|} \text{ (provided } \overrightarrow{|a|} \neq 0 \text{)}$$

Zero Vector OR Null Vector

A vector with a magnitude of zero, meaning it has identical initial and terminal points, is referred to as a zero vector, denoted by $\vec{O}$. It can have an arbitrary direction and any line as its line of support.

COLLINEAR VECTORS

Two vectors are considered collinear if their directed line segments are parallel, regardless of their specific directions. Collinear vectors are alternatively known as parallel vectors. If these vectors share the same direction, they are termed like vectors; otherwise, they are referred to as unlike vectors.

Symbolically, two non-zero vectors are collinear if and only if,

Where $l \in R$

$$\vec{a} = \lambda\vec{b} \Leftrightarrow \left(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right) = \lambda\left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right)$$

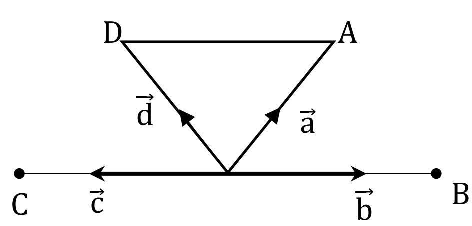
$$a_1 = \lambda b_1, a_2 = \lambda b_2, a_3 = \lambda b_3$$

$$= \frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3} (= \lambda)$$

Vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ are collinear if $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$

Co-initial vectors

Vectors that share the same initial point are referred to as co-initial vectors.



Coplanar Vectors (FOR COMPETITIVE EXAM)

A set of vectors is deemed coplanar if the lines of their supports are all parallel to the same plane.

Note:

The statement "Two vectors are always coplanar."
Coplanar vectors may have any directions or magnitude.

(i) Symbolically, two non-zero vectors $\vec{a}$ & $\vec{b}$ are collinear if and only if $\vec{a} = K\vec{b}$, where $K \in R$

(ii) If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ are two collinear vectors then $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$

(iii) If $\vec{a}$ & $\vec{b}$ are two non-zero, non-collinear vectors such that

$$x\vec{a} + y\vec{b} = 0$$

$$x = y = 0$$

Ex.1 Determine the unit vector of $\hat{i} - 2\hat{j} + 3\hat{k}$

Sol.

$$\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$$

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

$$|\vec{a}| = \sqrt{a_x{}^2 + a_y{}^2 + a_z{}^2}$$

$$|\vec{a}| = \sqrt{14}$$

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{1}{\sqrt{14}}\hat{i} - \frac{2}{\sqrt{14}}\hat{j} + \frac{3}{\sqrt{14}}\hat{k}$$

Ex.2 The value of λ when $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{b} = 8\hat{i} + \lambda\hat{j} + 4\hat{k}$ are parallel.

Sol. Since $\vec{a}$ & $\vec{b}$ are parallel

$$\frac{2}{8} = -\frac{3}{\lambda} = \frac{1}{4}$$

$$\lambda = -12$$

Ex.3 Determine the values of x & y for which the vectors are parallel.

$$\vec{a} = (x+2)\hat{i} - (x-y)\hat{j} + \hat{k}$$

$$\vec{b} = (x-1)\hat{i} + (2x+y)\hat{j} + 2\hat{k} \text{ are parallel.}$$

Sol. $\vec{a}$ and $\vec{b}$ are parallel if

$$\frac{x+2}{x-1} = \frac{y-x}{2x+y} = \frac{1}{2}$$

$$x = -5, y = -20$$

Ex.4 If $A \equiv (2\hat{i}+3\hat{j}), B \equiv (p\hat{i}+9\hat{j})$ and $C \equiv (\hat{i}-\hat{j})$ are collinear. Then find the value of p.

Sol.
$$\overrightarrow{AB} = (p-2)\hat{i}+6\hat{j}$$
$$\overrightarrow{AC} = -\hat{i}-4\hat{j}$$

Now A, B, C are collinear

$$\overrightarrow{AB} \parallel \overrightarrow{AC}$$
$$\frac{p-2}{-1} = \frac{6}{-4}$$
$$p = \frac{7}{2}$$

Ex.5 If $\vec{a}$ and $\vec{b}$ are vectors that are not collinear, determine the value of x for which the vectors: $\vec{\alpha} = (x-2)\vec{a}+\vec{b}$ and $\vec{\beta} = (3+2x)\vec{a}-2\vec{b}$ are collinear.

Sol. As the vectors $\vec{\alpha}$ and $\vec{\beta}$ are collinear.

There exist scalar l such that $\vec{\alpha} = \lambda\hat{\beta}$

$$(x-2)\vec{a}+\vec{b} = \lambda\{(3+2x)\vec{a}-2\vec{b}\}$$
$$(x-2-\lambda(3+2x))\vec{a}+(1+2\lambda)\vec{b} = 0$$
$$x-2-\lambda(3+2x) = 0 \quad \text{And} \quad 1+2\lambda = 0$$
$$x-2-\lambda(3+2x) = 0 \quad \text{And} \quad \lambda = -\frac{1}{2}$$
$$x-2+\frac{1}{2}(3+2x) = 0$$
$$4x - 1 = 0$$
$$x = \frac{1}{4}$$

EQUALITY OF TWO VECTORS (FOR COMPETITIVE EXAM)

Two vectors are considered equal if they possess:

(A) Equal length,
(B) Parallel or identical supports, and
(C) The same direction.

Note:

The components of two equal vectors, measured in any arbitrary direction, are identical.

I.e. If, $\vec{a} = a_1\hat{i}+a_2\hat{j}+a_3\hat{k}, \vec{b} = b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$

When $\hat{i}, \hat{j}$ & $\hat{k}$ represent the unit vectors along the coordinate axes, then

$$\vec{a} = \vec{b}$$
$$a_1 = b_1, a_2 = b_2, a_3 = b_3$$

Ex.6 Let $\vec{r} = 3\hat{i} + 2\hat{j} - 5\hat{k}, \vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{i} + 3\hat{j} - 2\hat{k}$ and $\vec{c} = -2\hat{i} + \hat{j} - 3\hat{k} \cdot \vec{r} = \lambda\vec{a} + \mu\vec{b} + v\vec{c}$ then find $\lambda + \mu + v$.

Sol.
$$3\hat{i} + 2\hat{j} - 5\hat{k}$$
$$= \lambda(2\hat{i} - \hat{j} + \hat{k}) + \mu(\hat{i} + 3\hat{j} - 2\hat{k}) + v(-2\hat{i} + \hat{j} - 3\hat{k})$$
$$= (2\lambda + \mu - 2v)\hat{i} + (-\lambda + 3\mu + v)\hat{j} + (\lambda - 2\mu - 3v)\hat{k}$$

Equating components of equal vectors

$$2\lambda + \mu - 2v = 3 \text{ ..... (i)}$$
$$-\lambda + 3\mu + v = 2 \text{ ..... (ii)}$$
$$\lambda - 2\mu - 3v = -5 \text{ ..... (iii)}$$

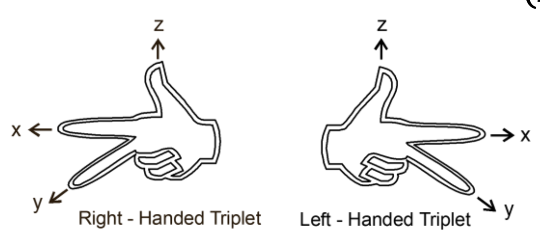
On solving (i),(ii) & (iii)

We get $\lambda = 3$, $\mu = 1$, $v = 2$

So, $\lambda + \mu + v = 6$

LEFT AND RIGHT–HANDED ORIENTATION (CONFIGURATIONS)

(FOR COMPETITIVE EXAM)



For each hand, consider the directions Ox, Oy, and Oz as depicted in the figure. Consequently, two rectangular coordinate systems emerge. Can they be made congruent? No, they cannot, due to the distinct orientations of the two hands. As a result, these two systems are distinct.

A rectangular coordinate system that can be made congruent with the system formed using the right hand (or left hand) is referred to as a right-handed (or left-handed) rectangular coordinate system.

Therefore, we have the following criterion to distinguish between these two systems based on the sense of rotation:

(A) If the rotation from Ox to Oy occurs in the counter clockwise direction, and Oz is oriented upwards (following the right-hand rule), then the system is considered right-handed.

(B) If the rotation from Ox to Oy is in the clockwise direction, and Oz is directed upward (following the left-hand rule), then the system is considered left-handed.

From now on, we will utilize the right-handed rectangular Cartesian coordinate system, also known as the ortho-normal system.