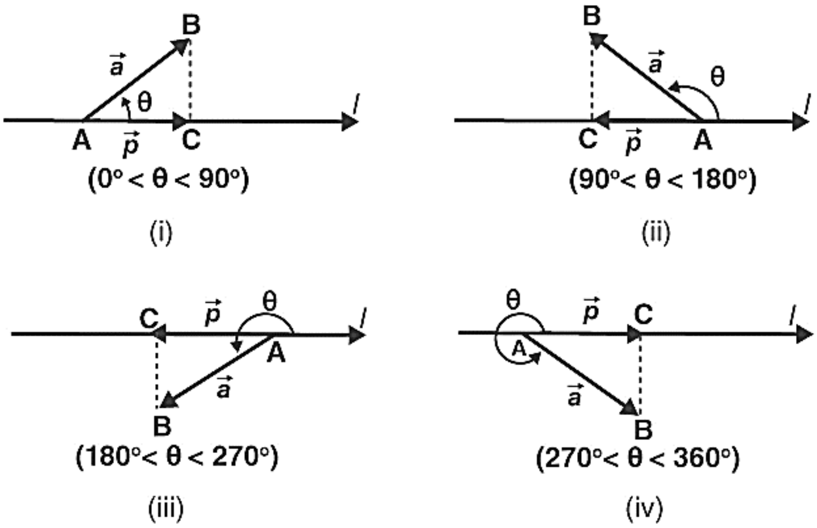
VECTOR ALGEBRA

PROJECTION OF A VECTOR ON A LINE

Assume that a vector $\overrightarrow{AB}$ forms an angle θ in the counter clockwise direction with a specified directed line, denoted as l, as illustrated in the following figure.



The projection of vector $\overrightarrow{AB}$ onto vector I results in a vector $\vec{p}$ (designated as such), with a magnitude of $|\overrightarrow{AB}|\ |\cos\theta|$. The direction of $\vec{p}$ aligns with or opposes that of line I, depending on whether $\cos\theta$ is positive or negative. This vector $\vec{p}$ is termed the projection vector, and its magnitude $|\vec{p}|$ is referred to as the projection of vector $|\overrightarrow{AB}|$ onto the directed line I.

As an illustration, in each of the figures below, the projection vector of vector $\overrightarrow{AB}$ onto line I is represented by vector $\overrightarrow{AC}$.

Observations:

(1) If $\hat{p}$ is the unit vector along a line I, then the projection of vector $\vec{a}$ onto the line I is expressed as $\vec{a}.\hat{p}$.

(2) The projection of vector $\vec{a}$ onto another vector $\vec{b}$ is determined by

$$\vec{a}.\hat{b},\ \text{or}\ \vec{a}\left(\frac{\vec{b}}{|\vec{b}|}\right),\ \text{or}\ \frac{1}{|\vec{b}|}(\vec{a}\cdot\vec{b})$$

(3) When $\theta = 0$, the projection vector of vector $\overrightarrow{AB}$ will be $\overrightarrow{AB}$ itself. Conversely, if $\theta = \pi$, the projection vectors of vector $\overrightarrow{AB}$ will be $\overrightarrow{BA}$

(4) If $\theta = \frac{\pi}{2}$ or $\theta = \frac{3\pi}{2}$, then the projection vector of $\overrightarrow{AB}$ will be zero vector.

Note:

If α, β and γ are the direction angles of a vector, $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ its direction cosines can be expressed as

$$\cos\alpha = \frac{\vec{a}\cdot\hat{i}}{|\vec{a}||\hat{i}|} = \frac{a_1}{|\vec{a}|}$$

$$\cos\beta = \frac{a_2}{|\vec{a}|}$$

$$\cos\gamma = \frac{a_3}{|\vec{a}|}$$

Additionally, observe that $|\vec{a}|\cos\alpha$, $|\vec{a}|\cos\beta$ and $|\vec{a}|\cos\gamma$ are the projections of vector $\vec{a}$ along the OX, OY, and OZ axes, denoted as a_1, a_2, and a_3, respectively. These scalar components a_1, a_2, and a_3 precisely represent the projections of vector $\vec{a}$ along the x-axis, y-axis, and z-axis. Furthermore, if vector $\vec{a}$ is a unit vector, it can be expressed in terms of its direction cosines as

$$\vec{a} = \cos\alpha\hat{i} + \cos\beta\hat{j} + \cos\gamma\hat{k}$$

Ex.1 Determine the angle between two vectors $\vec{a}$ and $\vec{b}$, each having magnitudes of 1 and 2, respectively. Additionally, it is known that the dot product of vectors $\vec{a}.\vec{b} = 1$.

Sol. We know that, $\vec{a}.\vec{b} = 1$

$$|\vec{a}| = 2 \text{ and } |\vec{b}| = 1$$

Hence, the angle θ between two vectors is,

$$\theta = \cos^{-1}\left(\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}\right)$$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$