VECTOR ALGEBRA

ADDITION OF VECTORS

ALGEBRA OF VECTORS

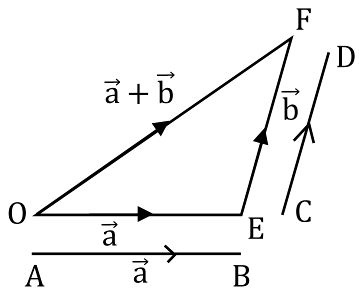
An algebra of vectors can be constructed, proving beneficial in the exploration of geometry, mechanics, and various other fields of applied mathematics.

(A) Addition of two vectors

Vectors possess both magnitude and direction, and as a result, their addition differs from the addition of real numbers.

Consider two vectors $\vec{a}$ and $\vec{b}$ in a plane, represented by $\overrightarrow{AB}$ and $\overrightarrow{CD}$. Their addition can be carried out in two different manners:

Triangle law of addition of vectors:



If two vectors can be expressed in magnitude and direction by the two sides of a triangle, taken consecutively, then the third side will represent their sum in the opposite order.

Consider a fixed point O in the vector plane. Draw a line segment $\overrightarrow{OE}$ from O, equal and parallel to $\overrightarrow{AB}$, representing the vector $\vec{A}$. Subsequently, from point E, draw a line segment $\overrightarrow{EF}$ equal and parallel to $\overrightarrow{CD}$ representing the vector $\vec{B}$.

The line segment $\overrightarrow{OF}$, formed by connecting O and F, represents the sum of vectors $\vec{A}$ and $\vec{B}$.

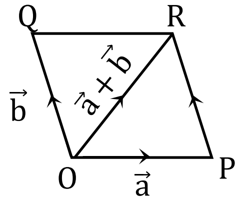
$$\overrightarrow{OE} + \overrightarrow{EF} = \overrightarrow{OF}$$

$$\vec{a} + \vec{b} = \overrightarrow{OF}$$

The process of adding two vectors using this approach is known as the Triangle Law of Vector Addition.

Parallelogram law of addition of vectors:

If two vectors are represented in magnitude and direction by the two consecutive sides of a parallelogram, then their sum will be represented by the diagonal passing through the co-initial point.



Consider vectors $\vec{a}$ and $\vec{b}$ originating from point O, represented by line segments. Now, complete the parallelogram OPRQ.

The vector represented by the diagonal OR will denote the sum of the vectors. .

$$\overrightarrow{OP}+\overrightarrow{OQ}=\overrightarrow{OR}$$

$$\vec{a}+\vec{b}=\overrightarrow{OR}$$

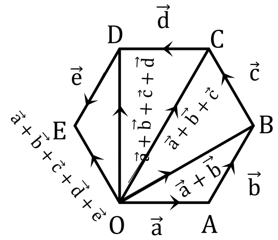
The process of adding two vectors using this technique is known as the Parallelogram Law of Vector Addition.

Properties of vector addition

1. If two vectors $\vec{a}$ and $\vec{b}$ are represented by $\overrightarrow{OA}$ and $\overrightarrow{OB}$, then their sum $\vec{a}+\vec{b}$ is a vector represented by $\overrightarrow{OC}$, where OC is the diagonal of the parallelogram OACB.
2. $\vec{a}+\vec{b}=\vec{b}+\vec{a}$ (Commutative)
3. $(\vec{a}+\vec{b})+\vec{c}=\vec{a}+(\vec{b}+\vec{c})$ (Associative)
4. $\vec{a}+\vec{0}=\vec{a}=\vec{0}+\vec{a}$ (Additive identity)
5. $\vec{a}+(-\vec{a})=\vec{0}=(-\vec{a})+\vec{a}$ (Additive inverse)
6. $|\vec{a}+\vec{b}|\le|\vec{a}|+|\vec{b}|$
7. $|\vec{a}-\vec{b}|\ge||\vec{a}|-|\vec{b}||$

Polygon law of vector Addition (Addition of more than two vectors)

The addition of more than two vectors is accomplished through the repetition of the Triangle Law.



Suppose we need to determine the sum of five vectors $\vec{a},\vec{b},\vec{c},\vec{d}$ and $\vec{e}$ .If these vectors are represented by line segments forming a polygon $\overrightarrow{OA},\overrightarrow{AB},\overrightarrow{BC},\overrightarrow{CD}$ and $\overrightarrow{DE}$ respectively, their sum will be denoted by $\overrightarrow{OE}$. This vector is represented by the remaining (last) side of the polygon OABCDE in reverse order. This can also be clarified as follows:

By triangle's law

$$\overrightarrow{OA}+\overrightarrow{AB}=\overrightarrow{OB} \qquad \text{Or} \qquad \vec{a}+\vec{b}=\overrightarrow{OB}$$

$$\overrightarrow{OB}+\vec{c}=\overrightarrow{OC} \qquad \text{Or} \qquad (\vec{a}+\vec{b})+\vec{c}=\overrightarrow{OC}$$

$$\overrightarrow{OC}+\vec{d}=\overrightarrow{OD} \qquad \text{Or} \qquad (\vec{a}+\vec{b}+\vec{c})+\vec{d}=\overrightarrow{OD}$$

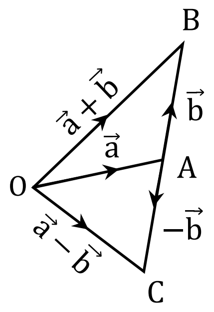
$$\overrightarrow{OD}+\vec{e}=\overrightarrow{OE} \qquad \text{Or} \qquad (\vec{a}+\vec{b}+\vec{c}+\vec{d})+\vec{e}=\overrightarrow{OE}$$

In this context, $\overrightarrow{\mathbf{OE}}$ is depicted as the line segment connecting the initial point o of the first vector $\vec{\mathbf{a}}$ and the terminal point of the last vector $\vec{\mathbf{e}}$. To determine the sum of more than two vectors using this approach, a polygon is constructed. Hence, this technique is referred to as the polygon law of addition. If the initial point of the first vector and the terminal point of the last vector coincide, the sum of the vectors will result in a null vector.

(B) Subtraction of Vectors

Previously, we discussed the vector $-\vec{\mathbf{b}}$, which has the same magnitude as vector $\vec{\mathbf{b}}$ but opposite direction. The subtraction of vector $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ is defined as the addition of $(-\vec{\mathbf{b}})$

It is written as follows: $\vec{a}-\vec{b}=\vec{a}+(-\vec{b})$



Geometrical representation

In the provided diagram, vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are depicted by $\overrightarrow{\mathbf{OA}}$ and $\overrightarrow{\mathbf{AB}}$, respectively. We extend the line AB in the opposite direction to point C, where AB = AC. The line segment $\overrightarrow{\mathbf{AC}}$ will represent the vector $\overrightarrow{-\mathbf{b}}$ By connecting points O and C, the vector represented by $\overrightarrow{\mathbf{OC}}$ is $\vec{\mathbf{a}}+(\overrightarrow{-\mathbf{b}})$. i.e. denotes the vector $\vec{\mathbf{a}}-\vec{\mathbf{b}}$.

Note:

(i) $\vec{a}-\vec{a}=\vec{a}+(-\vec{a})=\vec{0}$

(ii) $\vec{a}-\vec{b}\neq\vec{b}-\vec{a}$

Therefore, the subtraction of vectors does not adhere to the commutative law.

(iii) $\vec{a}-(\vec{b}-\vec{c})\neq(\vec{a}-\vec{b})-\vec{c}$

In other words, the subtraction of vectors does not follow the associative law.