THERMAL PROPERTIES OF MATTER

THERMAL EXPANSION

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The majority of materials exhibit an expansion in volume as their temperature rises. This phenomenon is particularly evident in devices such as liquid-in-tube thermometers, illustrated in Figure (a), where an increase in temperature causes the liquid to expand. Bimetallic strips, showcased in Figure (b), also demonstrate this thermal expansion as their shape is altered with changes in temperature. In practical applications, structures like bridges incorporate special joints and supports to accommodate the expansion resulting from rising temperatures.

Examples of thermal expansion can be observed in everyday scenarios, such as the cracking of a completely filled and tightly capped bottle of water when subjected to heat. Conversely, the lid of a metal jar may loosen when exposed to hot water. These instances underscore the importance and ubiquity of thermal expansion phenomena.

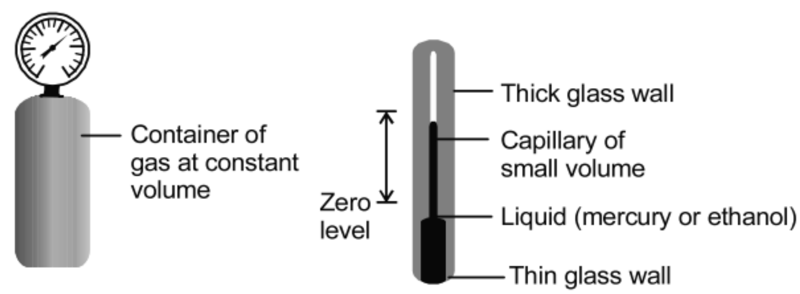

Fig.: (a) Changes in temperature cause the pressure of the gas to change, and mercury in the thermometer rise.

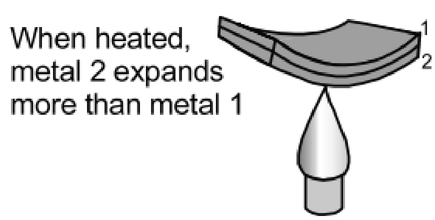

Fig.: (b)

Linear Expansion

Consider a rod composed of a specific material with an initial length L at an initial temperature T_0. When the temperature undergoes a change ΔT, the corresponding change in length is denoted by ΔL. Experimental observations indicate that for relatively small temperature changes (e.g., less than 100°C), ΔL is directly proportional to ΔT. If two rods, both constructed from the same material, experience identical temperature variations, but one is twice the length of the other, the change in length for the longer rod is twice as significant. Consequently, ΔL is also proportional to the initial length L_0. Introducing a material-dependent proportionality constant a, distinct for each material, this relationship can be expressed in the form of an equation.

The change in length ΔL of a material undergoing thermal expansion can be expressed by the equation:

$\Delta L = \alpha \cdot L0 \cdot \Delta T$ ..... (i)

Where:

- ΔL Is the change in length,
- α Is the coefficient of linear expansion,
- L_0 is the initial length,
- ΔT is the change in temperature.

For a body with an initial length L_0 at a temperature T_0, its length (L) at a temperature $T = T_0 + \Delta T$ can be represented as:

$L = L_0 + \Delta L = L_0 + \alpha \cdot L_0 \cdot \Delta T$

Simplifying this expression, we get:

$L = L_0(1 + \alpha\Delta T)$ ..... (ii)

The constant α, known as the coefficient of linear expansion, characterizes the thermal expansion properties of a specific material. The units of α are either K^{-1} or$(°C^{-1})$.

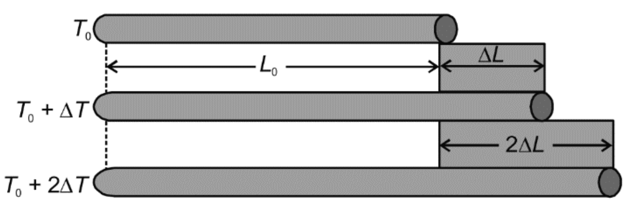

Fig.: (a) For moderate temperature changes, ΔL is directly proportional to ΔT

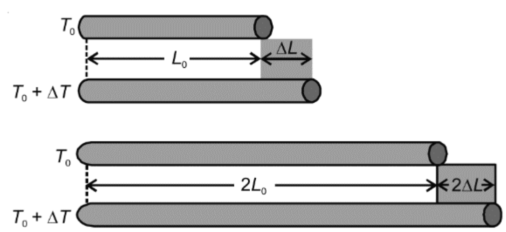

Fig.: (b) ΔL is also directly proportional to L_0

The direct proportionality described by Equation (i) is an approximation and is only accurate for relatively small temperature changes. In reality, the coefficient of linear expansion (α) for a specific material may exhibit some variation based on factors such as the initial temperature (T_0) and the magnitude of the temperature change ΔT. For the sake of simplicity, we will disregard these complexities in our current discussion.

Volume Expansion

Elevating the temperature also induces a rise in volume for both solid and liquid substances.

Experimental observations reveal that, for relatively modest temperature changes ΔT, less than about 100°C), the increase in volume ΔV is approximately proportional to both the temperature change ΔT and the initial volume(V_0). This relationship can be expressed by the equation:

$\Delta V = \beta \cdot V_0 \cdot \Delta T$ ..... (iii)

Here, the constant β characterizes the volume expansion properties specific to a material and is referred to as the coefficient of volume expansion. The units of β are K^{-1} or (°C^{-1}). Similar to linear expansion, the coefficient β may exhibit some variation with temperature. It's important to note that Equation (iii) provides an approximate relationship that holds true only for small temperature changes. For many substances, β tends to decrease at lower temperatures.

Relation between Volume Expansion and Linear Expansion

For solid materials, a straightforward connection exists between the volume expansion coefficient (β) and the linear expansion coefficient (α). To elucidate this relationship, let's contemplate a cube of material with a side length L and volume $V = L^3$. At the initial temperature, the respective values are L and V. When the temperature increases, the side length augments by ΔL, and the volume experiences an alteration ΔV described by the equation:

$$\Delta V = \frac{dL}{dV} \cdot \Delta L = 3L^2 \cdot \Delta L$$

Here, $\frac{dV}{dL}$ represents the rate of change of volume with respect to the change in linear dimension (L).

Now, we substitute L and V with their initial values L_0 and V_0. Utilizing Equation (i), we find ΔL as $\Delta L = \alpha\, L_0 \Delta T$. As $V = L^3$, this implies that ΔV can also be expressed as:

$$\Delta V = 3\alpha L \Delta L = 3\alpha V \Delta T$$

This expression aligns with the infinitesimal form of Equation (iii), i.e., $\Delta V = \beta V \Delta T$ only when ΔT is infinitesimally small. Consequently, we deduce that $\beta = 3\alpha$ (Equation iv).

To validate this relationship, one can verify it for various materials listed in the tables above. Similarly, the expansion in area is termed area expansion or superficial expansion, and the fractional change $\frac{\Delta A}{A}$ in area is given by $\frac{\Delta A}{A_0} = \beta \Delta T$, where β is called the coefficient of area expansion. It is related to α by the equation $\beta = 2\alpha$.

Example.

A surveyor employs a steel measuring tape with an exact length of 50.000 meters at a temperature of 20 °C. What will be its length on a warm summer day when the temperature reaches 35 °C? (Given: $\alpha\ steel = 1.2 \times 10^{-5}\ K^{-1}$)

Solution.

The change in temperature, denoted as $\Delta T = T - T_0$, is 15 °C. Utilizing equation (ii), the corresponding alterations in length,ΔL and the final length $L = L_0 + \Delta L$, can be determined as follows:

$$\Delta L = \alpha \cdot Lp. \Delta T = (1.2 \times 10^{-5} K^{-1})(50m)(15K) = 9 \times 10^{-3m} = 9mm$$

$$L = L_0 + \Delta L = 50m + 0.009m = 50.009m$$

Hence, the length at 35 degrees Celsius is 50.009 meters. This example illustrates that metals exhibit minimal expansion under moderate temperature variations. Even a metal baking pan in a 200 °C oven expands only slightly compared to its dimensions at room temperature.

Example.

A glass flask possessing a volume of 200 cm^3 is completely filled with mercury at a temperature of 20°C. The inquiry pertains to the amount of mercury that spills over when the temperature of the entire system is elevated to 100°C. The glass exhibits a coefficient of linear expansion amounting to $0.40 \times 10^{-5}\ K^{-1}$, and the cubical expansion coefficient of mercury is $18 \times 10^{-5} K^{-1}$.

Solution.

The coefficient of volume expansion for the glass is denoted as $Y_{glass} = 30_{glass} = 1.2 \times 10^{-5} K^{-1}$. The increase in volume of the glass flask is given by:

$$\Delta V_{glass} = Y_{glass} \cdot V_{glass}. \Delta T = (1.2 \times 10^{-5} K^{-1}) \cdot (200cm^3) \cdot (100°C - 20°C) = 0.19cm^3$$

Similarly, the increase in volume of the mercury is expressed as:

$$\Delta V_{mercury} = Y_{mercury} \cdot V_{mercury} \cdot \Delta T = (18 \times 10^{-5} K^{-1}).(200cm^3) \cdot (100°C - 20°C) = 2.9cm^3$$

The volume of mercury that overflows is then calculated by subtracting the increase in glass volume from the increase in mercury volume:

$$\Delta V_{mercury} - \Delta V_{glass} = 2.9cm^3 - 0.19cm^3 = 2.7cm^3$$