STRAIGHT LINES

VARIOUS FORMS OF THE EQUATION OF A LINE

Various Forms of the Equation of a straight Line

It was said earlier that every curve, in coordinate geometry, is represented by an equation which is a relation connecting x and y, the coordinates of any point which lies on that curve. In particular, any straight line has also an equation to represent it. It may be seen that to represent a straight line we need to be given two independent information; and depending upon the two, the equation also changes.

We have therefore the following forms of equations to straight lines:

(i) SLOPE ONE POINT FORM

Given that a line has a slope ($m = \tan\theta$) which gives the direction it may be noted that 'm' alone does not give the equation of the line and with the same slope there can be any number of straight lines all of which are parallel. Given that it passes through a given point (x_1, y_1). In this case the equation has the form.

$$\mathbf{y - y_1 = m(x - x_1)}$$

Ex.1 If a line has a slope $= \frac{1}{2}$ and passes through (– 1, 2); find its equation.

Sol. The equation of the line is

$$y - 2 = \frac{1}{2}\left(x - (-1)\right)$$

i.e., $2y - 4 = x + 1$

i.e., $x - 2y + 5 = 0$

(ii) Y-INTERCEPT FORM

Given the slope 'm' and the length 'c' (called the intercept) cut off on the y-axis by the line. In this case the form of the equation is

$$\mathbf{y = mx + c}$$

Ex.2 If a line has a slope $\frac{1}{2}$ and cuts off along the positive y-axis of length $\frac{5}{2}$ find the equation of the line.

Sol. $y = \frac{1}{2}x + \frac{5}{2}$

i.e., $2y = x + 5$

i.e., $x - 2y + 5 = 0$

(iii) TWO POINT FORM

Equation of a line passing through two points (x_1, y_1) and (x_2, y_2).

In this case the equation to the line is of the form

$$\frac{y-y_1}{x-x_1} = \frac{y_1-y_2}{x_1-x_2}$$

Where $x_1 \neq x_2$ and $\left(\frac{y_1-y_2}{x_1-x_2}\right)$ is the slope (m) of the line.

Ex.3 If a line passes through two points (1, 5) and (3, 7), find its equation.

Sol.
$$\frac{y-5}{x-1} = \frac{-7+5}{-3+1} = \frac{-2}{-2} = 1$$

i.e., $y - 5 = x - 1$

i.e., $x - y + 4 = 0$

(iv) SYMMETRIC FORM

If (x_1, y_1) is a given point on a straight line, (x, y) any point on the line, θ the inclination of the line and r the distance between the points (x, y) and (x_1, y_1) then

$$\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta}$$

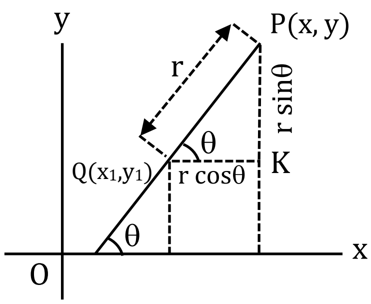
Let P (x, y) represents any point on the line.

$Q(x_1, y_1)$ is a given point, such that PQ = r.

Draw lines through Q and P parallel to QY, and draw QK parallel to OX.

From the triangle QKP.

$$QK = PQ \text{ COS } \theta$$



or $$x - x_1 = r\cos\theta$$

and $$\frac{x - x_1}{\cos\theta} = r$$

Also $$PK = PQ \sin\theta$$

or $$y - y_1 = r\sin\theta$$

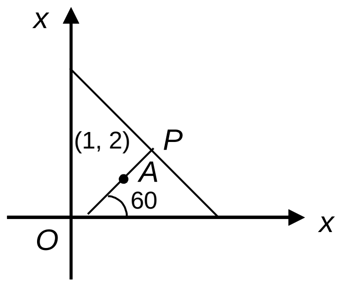
and $$\frac{y - y_1}{\sin\theta} = r$$

$\therefore$ $$\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$$

This is called the symmetric form of the equation to a line (This will be found very useful in cases where we have to deal with the length (r) of any portion of a line.)

Ex.4 A straight line passes through a point A (1, 2) and makes an angle 60° with the x-axis. This line intersects the line x + y = 6 at the point P. Find AP.

Sol. Let required line be



$$\frac{x-1}{\cos 60°} = \frac{y-2}{\sin 6°} = r(\equiv AP)$$

$\therefore$ P is (1 + r cos 60°, 2 + r sin 60°)

which lies on x + y = 6

The condition is

$$\left(1 + \frac{r}{2}\right) + \left(2 + r\frac{\sqrt{3}}{2}\right) = 6$$

Or $$r\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) = 3$$

$\therefore$ $$r = \frac{6}{1+\sqrt{3}} = \frac{6\left(1-\sqrt{3}\right)}{\left(1+\sqrt{3}\right)\left(1-\sqrt{3}\right)}$$

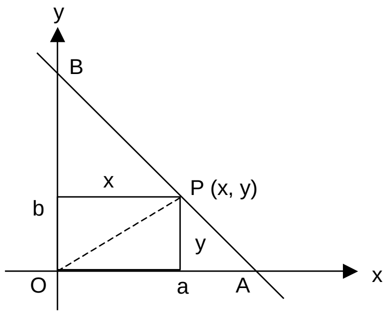
$$r = \frac{6\left(1-\sqrt{3}\right)}{\left(1+\sqrt{3}\right)\left(1-\sqrt{3}\right)} = \frac{6\left(1-\sqrt{3}\right)}{-2}$$

$$r = \ r = 3\left(\sqrt{3}-1\right).$$

(v) INTERCEPT FORM

Let the line make intercepts a and b on the axes of x and y respectively.

Let P(x, y) represent any point on the line. Join OP.



We have,

Area of triangle OAB = area of triangle OAP + area of triangle OPB

i.e., $$\frac{1}{2}ab = \frac{1}{2}ay + \frac{1}{2}bx$$

or $$\frac{x}{a} + \frac{y}{b} = 1$$

This is called the intercept form of the equation of straight line.

Ex.5 Find the equation of the straight line, which passes through the point (3, 4) and whose intercept on y-axis is twice that on x-axis.

Sol. Let the equation of the line be

$$\frac{x}{a} + \frac{y}{b} = 1 \qquad ...(i)$$

According to the question b = 2a

$\therefore$ from (i) equation of line will become

$$\frac{x}{a} + \frac{y}{2a} = 1$$

or $$2x + y = 2a \qquad ....(ii)$$

Since line (ii) passes through the point (3, 4)

$\therefore$ $$2 \times 3 + 4 = 2a$$

$\therefore$ $$a = 5$$

$\therefore$ from (ii), equation of required line will be

$$2x + y = 10.$$

(vi) NORMAL FORM

If p is the length of the perpendicular from the origin upon a straight line, and that α is the angle the perpendicular makes with the axis of x, equation of the straight line can be obtained as follows:

Let PQ be the straight line, OQ (= p) the perpendicular drawn to it from the origin O, and $\angle QOX = \alpha$

Draw the ordinate PN also draw NR perpendicular to OQ, and PM perpendicular to RN

We have $\angle PNM = 90^\circ - \angle RNO = \alpha$

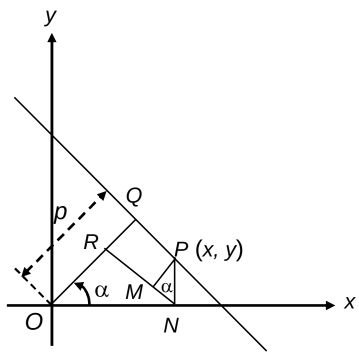
$\therefore$ $P = OQ = OR + RQ = OR + PM$

$= ON \cos\alpha + PN \sin\alpha$

or $p = x\cos\alpha + y\sin\alpha$

$\therefore$ $x\cos\alpha + y\sin\alpha = p$ is the required equation.

This is called the perpendicular form.



Alternatively,

Suppose $\frac{x}{a} + \frac{y}{b} = 1$ is the equation of the straight line

In this case, $a = \frac{p}{\cos\alpha}, b = \frac{p}{\sin\alpha}$

$\therefore$ by substitution, the equation becomes

$$\frac{x\cos\alpha}{p} + \frac{y\sin\alpha}{p} = 1$$

or $\mathbf{x\cos\alpha + y\sin\alpha = p}$

This is known as the normal form of the equation to a straight line.

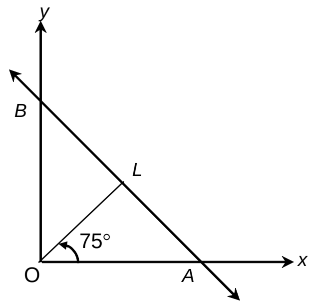
Ex.6 Find the equation of the straight line upon which the length of perpendicular from origin is $3\sqrt{2}$ units and this perpendicular makes an angle of 75° with the positive direction of x-axis.

Sol. Let AB be the required line and OL be perpendicular to it.

Given $OL = 3\sqrt{2}$ and $\angle LOA = 75^\circ$

$\therefore$ equation of line AB will be

[Normal form] $x\cos 75^\circ + y\sin 75^\circ = 3\sqrt{2}$ ...(i)



Now $\cos 75° = \cos(30° + 45°)$

$$= \frac{\sqrt{3}}{2}.\frac{1}{\sqrt{2}} - \frac{1}{2}.\frac{1}{\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

and $\sin 75° = \sin(30° + 45°)$

$$= \sin30°\cos45° + \cos30°\sin45°$$

$$= \frac{1}{2}.\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}.\frac{1}{\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$\therefore$ from (i) equation of line AB is

$$x\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right) + y\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right) = 3\sqrt{2}$$

or $$\left(\sqrt{3}-1\right)x + \left(\sqrt{3}+1\right)y - 12 = 0$$

(vii) GENERAL FORM

Any linear (1^{st} degree in x and y) equation of the form

$$Ax + By + C = 0$$

represents a straight line.

The straight line in this form, has

(a) $$\text{slope} = m = -\frac{A}{B} = -\frac{\text{coefficient of } x}{\text{coefficient of } y}$$

(b) $$x \text{ intercept} = -\frac{C}{A}$$

And $$y \text{ intercept} = -\frac{C}{B}$$

Ex.7 Find the value of k so that the straight line $2x + 3y + 4 + k(6x - y + 12) = 0$ and $7x + 5y - 4 = 0$ are perpendicular to each other.

Sol. Given lines are

$$(2 + 6k)x + (3 - k)y + 4 + 12k = 0 \quad \text{...(i)}$$

and $$7x + 5y - 4 = 0 \quad \text{...(ii)}$$

slope of line (i), $$m_1 = -\frac{2+6k}{3-k} = \frac{2+6k}{k-3}$$

and slope of line (ii), $$m_2 = -\frac{7}{5}$$

Since line (i) is perpendicular to line (ii)

$\therefore \qquad \left(\frac{2+6k}{k-3}\right)\left(-\frac{7}{5}\right) = -1$

or $\qquad (2 + 6k)\, 7 = 5\, (k - 3)$

or $\qquad 14 + 42k = 5k - 15$

or $\qquad 37k = -29$

or $\qquad k = -\frac{29}{37}$