STRAIGHT LINES

SLOPE OF A LINE

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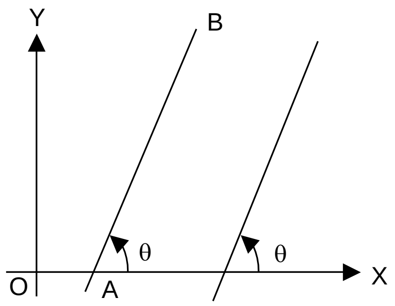
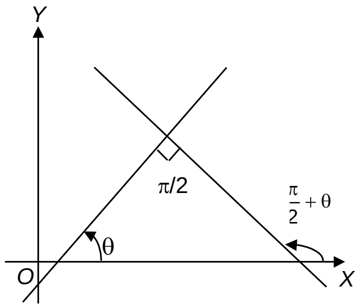




If a straight line AB makes an angle θ with the positive direction of the x-axis,

tan θ is called the slope or gradient of the straight line and is usually denoted by the letter 'm'.

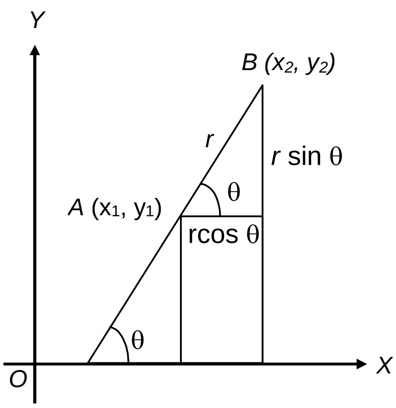
It follows that

(i) if two lines are parallel, their slopes are equal, for the lines must be equally inclined to the positive direction of the x-axis.

(ii) if two lines are perpendicular the product of their slopes is –1, for if one line is inclined at an angle θ to the x-axis, the other must be inclined at $\frac{\pi}{2}+\theta$, hence their slopes are tan θ and tan $\left(\frac{\pi}{2}+\theta\right)$, i.e., tan θ and –cot θ.

∴ The product is –1.

(i) TO FIND THE SLOPE OF THE LINE JOINING ANY TWO POINTS (x_1, y_1) AND (x_2, y_2)



Let the segment joining of the points be of length r and let the line be inclined to the x-axis at angle θ.

$\therefore$ $$r\cos\theta = x_2 - x_1$$

$$r\sin\theta = y_2 - y_1$$

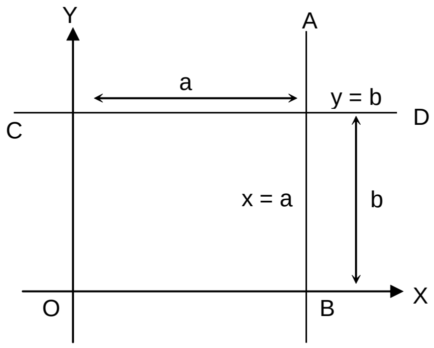
and therefore $$\tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$$

and tan θ is the required slope. The expression for the slope is, therefore,

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{\text{difference of the ordinates of the two points}}{\text{difference of the abscissa}}$$

(ii) LINES PARALLEL TO THE CO ORDINATE AXES

Let the line AB be parallel to the Y-axis and at a distance 'a' from it. Every point on AB will have its abscissa 'a', and hence the equation of AB is x = a. By putting a = 0, we deduce that the equation of the y-axis is x = 0.



Similarly, the equation of the straight line CD parallel to the X-axis and at a distance 'b' from it is y = b. By putting b = 0 we deduce that the equation to the X-axis is

y = 0

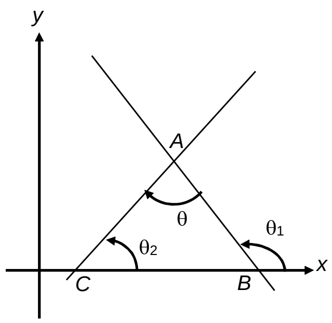
(iii) ANGLE BETWEEN TWO GIVEN STRAIGHT LINES

Let AB, AC have slopes m_1, m_2 and be inclined to the X-axis at θ_1, θ_2.

Then $$\angle = \theta_1 - \theta_2 = \theta$$

$\therefore$ $$\tan\angle CAB = \frac{\tan\theta_1 - \tan\theta_2}{1 + \tan\theta_1 \tan\theta_2}$$

i.e., $$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$$



where θ is the angle between AB and AC

Note: It is customary to take θ, as the acute angle between the two lines, and hence mostly one can take the above formula as $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

If the lines are parallel,

$$\tan\theta = 0, \quad \text{since } \theta = 0$$

$\therefore$ $m_1 - m_2 = 0$

$\therefore$ $\mathbf{m_1 = m_2}$

and **if the lines are perpendicular,** tan θ is not defined, since $\theta = \frac{\pi}{2}$, and

therefore $m_1 m_2 + 1 = 0$

$\therefore$ $m_1 m_2 = -1.$

Ex.1 Find the acute angle between the two lines with slopes $\frac{1}{5}$ and $\frac{3}{2}$.

Sol. If the angle between the lines is θ,

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\tan\theta = \left|\frac{\frac{1}{5} - \frac{3}{2}}{1 + \left(\frac{1}{5}\right) x \left(\frac{3}{2}\right)}\right| = |-1| = 1$$

Therefore the angle is 45°.