STRAIGHT LINES

DISTANCE OF A POINT FROM A LINE

Distance of a Point from a Line

The perpendicular distance of a point from a line can be obtained when the equation of the line and the coordinates of the point are given.

Case I :

Let us first derive the formula for this purpose when the equation of the line is given in normal form.

Let the equation of the line l in normal form be

$$x \cos \alpha + y \sin \alpha = p,$$

when α is the angle made by the perpendicular from origin to the line with positive direction of x-axis and p is the length of this perpendicular. Let $P(x_1, y_1)$ be the point, not on the line l. Let the perpendicular drawn from the point P to the line l be PM and PM = d. Point P is assumed to lie on opposite side of the line l from the origin O. Draw a line l′ parallel to the line l through the point P. Let ON be perpendicular from the origin to the line l which meets the line l′ in point R.

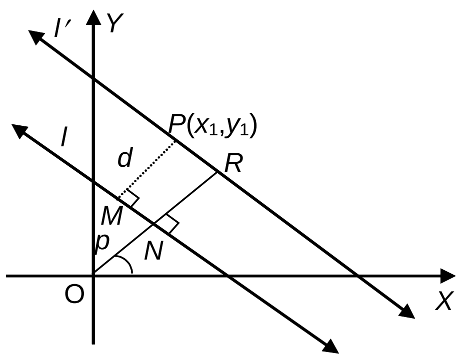
Obviously $ON = p$ and $\angle XON = \alpha$

Also, from the Figure, we note that

$$OR = ON + NR = p + MP$$

Therefore, length of the perpendicular from the origin to the line l′ is

$$OR = p + d$$



and the angle made by the perpendicular OR with positive direction of x-axis is α. Hence, the equation of the line l′ in normal form is

$$x \cos \alpha + y \sin \alpha = p + d$$

Since the line l′ passes through the point P, the coordinates (x_1, y_1) of the point P should satisfy the equation of the line l′ giving

$$x_1 \cos \alpha + y_1 \sin \alpha = p + d,$$

or $$d = x_1 \cos \alpha + y_1 \sin \alpha - p .$$

The length of a segment is always non–negative. Therefore, we take the absolute value of the RHS,

i.e., $d = | x_1 \cos \alpha + y_1 \sin \alpha - p |$.

Thus, the length of the perpendicular is the absolute value of the result obtained by substituting the coordinates of point P in the expression $x \cos \alpha + y \sin \alpha - p$

Case II:

Let the equation of the line be

$$Ax + By + C = 0$$

Reducing the general equation to the normal form, we have

$$\pm\frac{A}{\sqrt{A^2+B^2}}x \pm \frac{B}{\sqrt{A^2+B^2}}y = \pm\frac{C}{\sqrt{A^2+B^2}}$$

Where sign is taken + or – so that the RHS is positive.

(a) When $C < 0$

In this case the normal form of the equation of the line / becomes

$$\frac{A}{\sqrt{A^2+B^2}}x + \frac{B}{\sqrt{A^2+B^2}}y = -\frac{C}{\sqrt{A^2+B^2}}$$

Now, from the result of case (I), the length of the perpendicular segment drawn from the point $P(x_1, y_1)$ to the line (II) is

$$d = \left| \frac{A}{\sqrt{A^2+B^2}}x_1 + \frac{B}{\sqrt{A^2+B^2}}y_1 + \frac{C}{\sqrt{A^2+B^2}} \right|$$

i.e., $$d = \left| \frac{Ax_1 + By_1 + C}{\sqrt{A^2+B^2}} \right|$$

(b) When $C > 0$

In this case the normal form of the equation (II) becomes

$$-\frac{A}{\sqrt{A^2+B^2}}x - \frac{B}{\sqrt{A^2+B^2}}y = \frac{C}{\sqrt{A^2+B^2}}$$

Again, by the result of case (I), the distance d is

$$d = \left| \frac{-Ax_1 - By_1 - C}{\sqrt{A^2 + B^2}} \right|$$

or $$d = \left| \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}} \right|$$

The perpendicular distance of (x_1, y_1) from the line $ax + by + c = 0$ is given by

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

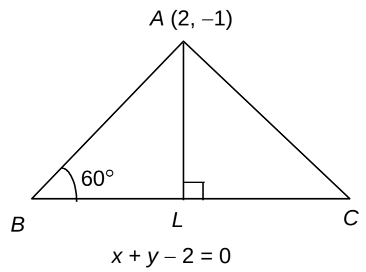
So, Perpendicular distance from origin is $\frac{|c|}{\sqrt{a^2 + b^2}}$.

Ex.1 The equation of the base of an equilateral triangle is $x + y = 2$ and the vertex is $(2, -1)$. Find the length of side of the triangle.

Sol. Equation of side BC is

$$x + y - 2 = 0 \qquad \text{...(i)}$$

$$A \equiv (2, -1)$$



Now $$AL = \left| \frac{2 - 1 - 2}{\sqrt{1^2 + 1^2}} \right| = \frac{1}{\sqrt{2}}$$

From ΔABL, $$\sin 60° = \frac{AL}{AB}$$

$\therefore$ $$\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{2}AB} \text{ or } AB = \sqrt{\frac{2}{3}}$$

Distance between parallel lines

The distance between $ax + by + c = 0$ and $ax + by + c' = 0$ is given by,

$$d = \frac{|c - c'|}{\sqrt{a^2 + b^2}}$$

Make sure that coefficients of x and y are same in both the lines before applying the formula.

Ex.2 Find the distance between the lines $5x + 12y + 40 = 0$ and $10x + 24y - 25 = 0$.

Sol. Here, coefficients of x and y are not the same in both the equations.

So, we write them as

$$5x + 12y + 40 = 0$$

$$5x + 12y - \frac{25}{2} = 0$$

Now, distance between them $= \dfrac{\left|40-\left(-\dfrac{25}{2}\right)\right|}{\sqrt{(5)^2+(12)^2}}$

$$= \frac{105}{2(13)} = \frac{105}{26}$$