# BINOMIAL THEOREM

## INTRODUCTION OF BINOMIAL THEOREM

### BINOMIAL EXPRESSION

Any algebraic expression which contains two dissimilar terms is called **Binomial expression.**

**For example:** $x-y,\ xy+\frac{1}{x},\ \frac{1}{z}-1,\ \frac{1}{(x-y)^{\frac{1}{3}}}+3$ etc.

**Terminology Used in Binomial Theorem** (For competitive exam)

**Factorial notation :** $\lfloor n$ or n! is pronounced as factorial n and is defined as

$$n! = \begin{cases} n(n-1)(n-2)......3.2.1 & \text{if } n \in N \\ 1 & \text{if } n = 0 \end{cases}$$

$$\text{v} \qquad n! = n\,.\,(n-1)! \quad ; \quad n \in N$$

### BINOMIAL THEOREM

The formula by which any positive integral power of a Binomial expression can be expanded in the form of a series is known as **BINOMIAL THEOREM.**

If x, y R and nN, then :

$$(x+y)^n = {}^nC_0x^n + n_{C_1}x^{n-1}y + {}^nC_2x^{n-2}y^2 + \ldots\ldots + {}^nC_1x^{n-r}y^r + \ldots\ldots + {}^nC_ny_n^n$$

This theorem can be proved by induction.

**Note:**

(a) The number of terms in the expansion is ( n+1) i.e. one more than the index.

(b) The sum of the indices of x & y in each term is n.

(c) The Binomial coefficients of the terms $({}^nC_0, {}^nC_1 \ldots\ldots)$ equidistant from the beginning and the end are equal. i.e. ${}^nC_p = {}^nC_{n-r}$

(d) Symbol ${}^nC_r$ can also be denoted by $\binom{n}{r}$, $C(n,r)$ or. .

The coefficient of $x^r$ in$(1+x)^n = n_{C_r}$ & that in $(1-x)^n = (-1)^r \cdot {}^{n_r}$

**Some Important Expansions :**

(i) $(1+x)^n_m = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + \ldots\ldots. {}^nC_nx^n$

(ii) $(1-x)^n = {}^nC_0 - {}^nC_1x + {}^nC_2x^2 + \ldots\ldots. + (-1)^n \cdot {}^nC_nx^n$

**Ex.** Expand the following Binomials:

(i) $(x - 3)^5$ (ii) $\left(1-\frac{3x^2}{2}\right)^4$

**Sol.** (i) $(x-3)^5$

$$ {}^5C_0x^5 + {}^5C_1x^4(-3)^1 + {}^5C_2x^3(-3)^2 + {}^5C_3x^2(-3)^3 + {}^5C_4x(-3)^4 + {}^5C_5(-3)^5 $$

$$ x^5 - 15x^4 + 90x^3 - 270x^2 + 405x - 243 $$

(ii) $\left(1-\frac{3x^2}{2}\right)^4$

$$ C_0^4 + C_1^4\left(\frac{-3x^2}{2}\right) + C_2^4\left(\frac{-3x^2}{2}\right)^2 + C_3^4\left(\frac{-3x^2}{2}\right)^3 + C_4^4\left(\frac{-3x^2}{2}\right)^4 $$

$$ = 1 - 6x^2 + \frac{27}{2}x^4 - \frac{27}{2}x^6 + \frac{81}{16}x^8 $$

**Ex.** Find the value of $\frac{\left(18^3 + 7^3 + 3\times18\times7\times25\right)}{3^6 + 6\times243\times2 + 15\times81\times4 + 20\times27\times8 + 15\times9\times16 + 6\times3\times32 + 64}$

**Sol.** The numerator is of the form

$$ a^3 + b^3 + 3ab(a+b) = (a+b)^3 $$

Where, $a = 18$ and $b = 7$

$\therefore$ $N^r = (18 + 7)^3 = (25)^3$

Denominator can be written as

$\therefore$ $\frac{Nr}{Dr} = \frac{(25)^3}{(25)^3} = 1$