PROBABILITY

EVENTS

4 **EVENT**

An Event, Which Is A Subset Of The Sample Space, Represents A Set Of Specific Outcomes From A Random Experiment.

- **Simple event**

  Every individual sample point within the sample space is referred to as an elementary event or simple event.

  For example The event of obtaining a head in a coin toss is a simple event.

- **Sure event**

  The set that includes all sample points is a certain event. For example, in the throw of a die, the event of obtaining a natural number less than 7 is a certain event.

- **Null event**

  The set that does not include any sample points.

- **Mixed/compound event**

  A subset of the sample space S that comprises more than one element is referred to as a composite event or a mixed event.

- **Compliment of an event**

  Consider S as the sample space and E as an event. The complement of event E, denoted as E^c or $\overline{E}$, is a subset that includes all sample points in S not present in E. It signifies the non-occurrence of event E.

5 **ALGEBRA OF EVENTS**

In the context of fundamental probability laws, we require the following concepts and facts regarding events (subsets) A, B, C, … of a given sample space S. The union $A \cup B$ of A and B includes all points in either A or B or both. The intersection $A \cap B$ of A and B comprises all points that belong to both A and B. If A and B have no common points, we express it as

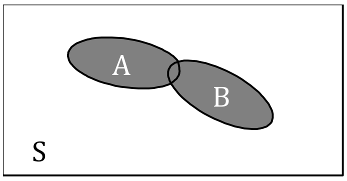
$$A \cap B = \phi$$

Where ϕ is the empty set (a set with no elements), and we refer to A and B as mutually exclusive (or disjoint) because the occurrence of A excludes that of B, and vice versa. For example, if a die turns up an odd number in a trial, it cannot turn up an even number in the same trial. Similarly, a coin cannot show both Head and Tail at the same time.

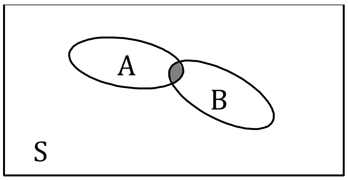
The complement A^c of A comprises all the points in S that are not in A. Thus,

$$A \cap A^C = \phi, \quad A \cup A^C = S$$

The use and understanding of events can be depicted and enhanced through Venn diagrams, which illustrate unions, intersections, and complements, as depicted in the figure.



Union $A \cup B$



Intersection $A \cap B$

Venn diagrams illustrate two events, A and B, within a sample space S, along with their union ($A \cup B$) and intersection ($A \cap B$) depicted in colour. The concepts of union and intersection extend similarly for more events.

$$\bigcup_{j=1}^{m} A_j = A_1 \cup A_2 \cup \ldots\ldots \cup A_m$$

The union of events $A_1, \ldots, Am$ comprises all points that belong to at least one of the A_j. Similarly, for the union $A_1 \cup A_2 \cup \ldots$ of an infinite number of subsets $A_1, A_2, \ldots$ of an infinite sample space S (meaning S contains an infinite number of points). The intersection

$$\bigcap_{j=1}^{m} A_j = A_1 \cap A_2 \cap \ldots\ldots \cap A_m$$

The intersection of $A_1, \ldots, Am$ consists of the points in S that are common to each of these events. Similarly, for the intersection $A_1 \cap A_2 \cap \ldots$ of an infinite number of subsets of S.

6 EQUALLY LIKELY EVENTS

The events are considered equally likely if there is no expectation that one of them will occur over the other.

For example

In the toss of a fair coin, the likelihood of a head or a tail occurring is equal. Therefore, the events of a head appearing and a tail appearing are equally likely.

7 MUTUALLY EXCLUSIVE EVENTS

A collection of events is considered mutually exclusive if the occurrence of one event prevents the occurrence of any other event.

For example:

- In the roll of a die, the event of obtaining an even number and the event of obtaining an odd number are mutually exclusive.
- In the toss of a fair coin, the occurrence of a head or a tail is mutually exclusive.

8 EXHAUSTIVE EVENTS

A set of events is exhaustive if the performance of the experiment results in occurrence of at least one of them.

For example:

- In the roll of a die, the event of obtaining an even number and the event of obtaining an odd number are exhaustive.
- In the toss of a fair coin, the occurrence of a head or a tail is exhaustive.

Exhaustive events cover the whole of the sample space. Their union is equal to S.

Definition of probability with discrete sample space

If the sample space S of an experiment comprises a finite number of outcomes (points) that are equally likely, then the probability of event A occurring is

$$P(A) = \frac{\text{Number of sample points in A}}{\text{Number of sample points in S}}$$

$$P(A) = \frac{n(A)}{n(S)}$$

In particular P(S) = 1 and $0 \le P(A) \le 1$.

Ex.1 Out of a set of 100 items, ten are defective. What is the probability that three out of any four chosen items are defective?

Sol. Probability $= \frac{{}^{90}C_1 \, . \, {}^{10}C_3}{{}^{100}C_4} = \frac{144}{52283}$

Ex.2 Seven individuals need to be seated on one side of a straight table. What is the probability that two specific persons will be seated next to each other?

Sol. Total number of ways of 7 persons being seated is ${}^{7}P_{7} = \underline{|7}$ ways

If two are to be seated next to each other, treat them as one unit – and this one unit with the remaining 5 can be seated in $\underline{|6}$ ways – and in each one of these $\underline{|6}$ ways the two persons can be interchanged in 2 ways.

$\therefore$ $$\text{Probability} = \frac{2\underline{|6}}{\underline{|7}} = \frac{2}{7}$$