INTRODUCTION TO THREE DIMENSIONAL GEOMETRY

SECTION FORMULA

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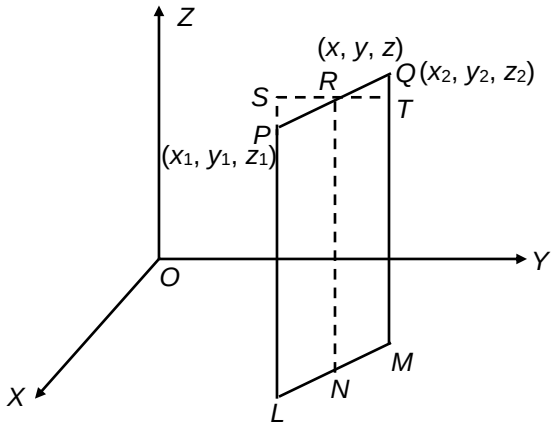
To find the coordinates of a point R which divides PQ internally in the ratio m : n, drop perpendicular from P, R and Q on the xy plane. Let the foot of perpendiculars on the xy plane be denoted as L, N and M.

$\because$ ΔPRS is similar to ΔRQT.

$\therefore$ the corresponding sides are proportional.

$\therefore$ $$\frac{PR}{RQ} = \frac{m}{n} = \frac{SP}{QT} = \frac{SR}{RT}$$



Now, $$SP = (SL - PL) = (RN - PL) = (z - z_1)$$

$$QT = (QM - TM) = (QM - RN) = (z_2 - z)$$

$\therefore$ $$\frac{m}{n} = \frac{(z - z_1)}{(z_2 - z)}$$

$\therefore$ $$(mz_2 - mz) = nz - nz_1$$

$$z(m + n) = mz_2 + nz_1$$

$$z = \frac{(mz_2 + nz_1)}{(m + n)}$$

Similarly $y = \frac{(my_2 + ny_1)}{(m+n)}$, $x = \frac{(mx_2 + nx_1)}{(m+n)}$

$\therefore$ The coordinates of point R are

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n} \right)$$

Similarly if R divides PQ externally in the ratio of m : n, then the coordinates of R are

$$\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n} \right)$$

Ex.1 Find the distance between the of points A(2, –1, 3) and B(–2, –1, 3).

Sol. Distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

$AB = \sqrt{(2+2)^2 + (-1+1)^2 + (3-3)^2} = 4$ units

Ex.2 Show that the points (2, –3, 4), (–1, 2, 1) and $\left(0, \frac{1}{3}, 2\right)$ are collinear and find the ratio in which the third point divides the line joining first two.

Sol. Let A denote the point (2, –3, 4) and B the point (–1, 2, 1)

Let C divides the line joining A and B in the ratio of $\lambda : 1$

$\therefore$ co-ordinates of C are

$$\left(\frac{2-\lambda}{\lambda+1}, \frac{-3+2\lambda}{\lambda+1}, \frac{4+\lambda}{\lambda+1} \right) \equiv \left(0, \frac{1}{3}, 2 \right) \quad \{\text{given}\}$$

$\therefore$ $\frac{2-\lambda}{\lambda+1} = 0$

$\Rightarrow$ $\lambda = 2$

and $\frac{-3+2\lambda}{\lambda+1} = \frac{1}{3}$

$\Rightarrow$ $-9 + 6\lambda = \lambda + 1$

$\Rightarrow$ $5\lambda = 10$

$\Rightarrow$ $\lambda = 2$

Also $\frac{4+\lambda}{\lambda+1} = 2$

$\Rightarrow$ $4 + \lambda = 2\lambda + 2$

$\Rightarrow \qquad \lambda = 2$

From each equation we get the value of λ as 2.

$\therefore$ The point A, C and B are collinear.

$\therefore$ The point C divides the line joining A and B internally in the ratio of 2 : 1.

Important formulas/ points

- Distance formula $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$
- Section formula $x=\frac{mx_2+nx_1}{m+n}, y=\frac{my_2+ny_1}{m+n}, z=\frac{mz_2+nz_1}{m+n}$
- For internal division m: n is positive.
- For external division m: n is negative.