WAVES

BEATS

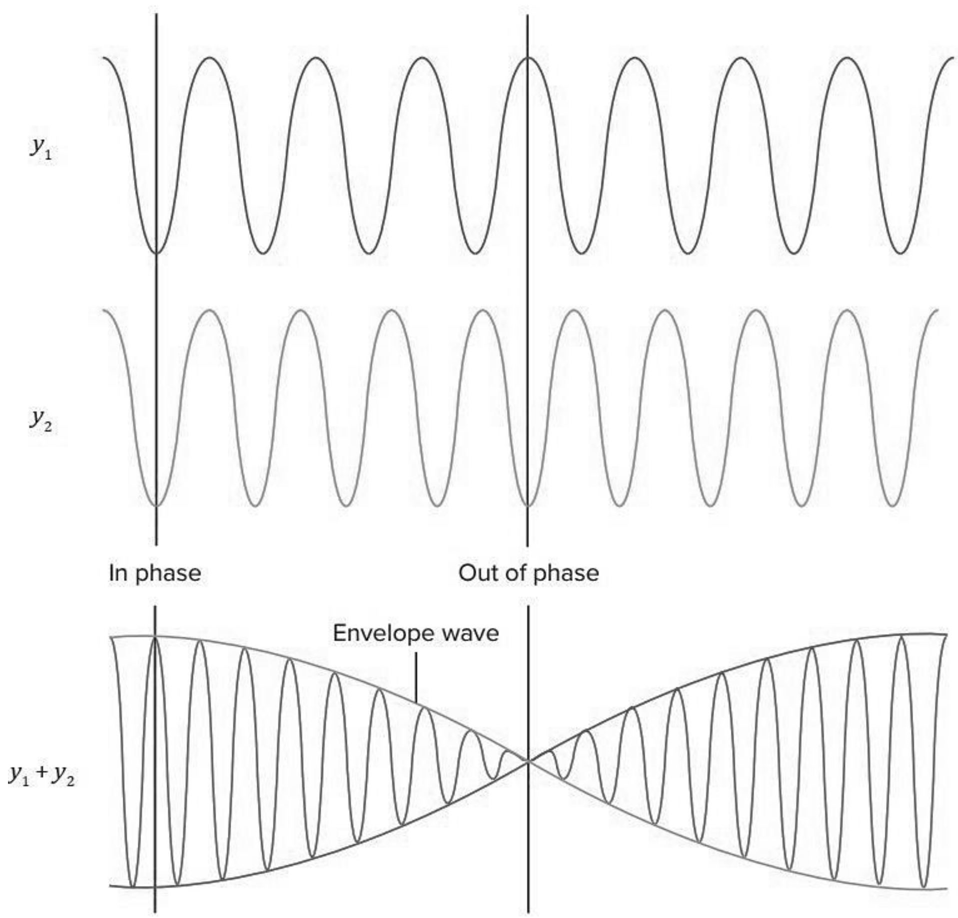
BEATS

Let's examine the combination of two sound waves that don't sync up perfectly and have different angular frequencies, denoted as ω1 and ω2. Their movement is described as follows:

$$y_1 = A_1 \sin(\omega_1 t + k_1 x_1 + \phi_1), \text{ and}$$
$$y_2 = A_2 \sin(\omega_2 t + k_2 x_2 + \phi_2)$$

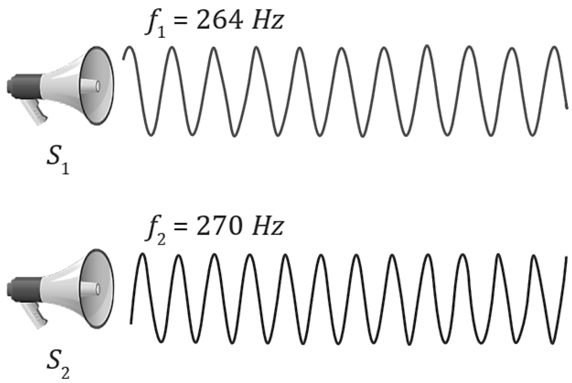
If we make the phase difference caused by the distance $(k_1 x_1 - k_2 x_2)$ and the starting phase $(\omega_1 - \omega_2)$ between the waves equal to zero using a certain setup, then the size of the combined wave goes up and down in a pattern, sort of like the sound of an ambulance siren getting louder and softer.

The periodic variations of the intensity of sound when two sound waves of slightly different frequencies interfere are known as beats.

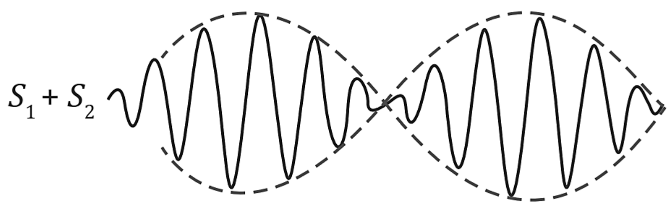


Beat Frequency

Think about two sound sources that don't match up in frequency, and they produce sounds at 264 Hz (f_1) and 270 Hz (f_2). It's important to notice that the gap between these two frequencies is quite small, only 6 Hz.

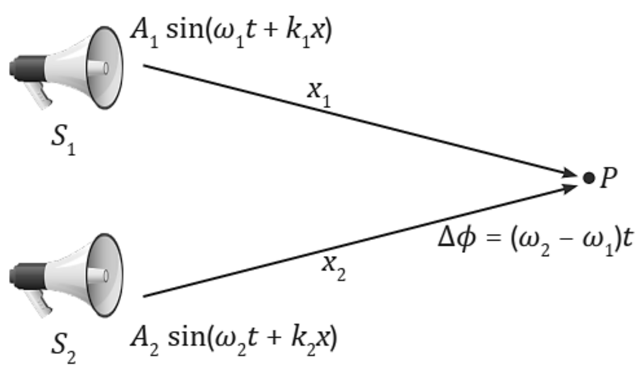


When we combine these sounds, we can see that the loud and quiet moments in the resulting wave appear, and we call these patterns beats, as you can see in the picture.



Let's contemplate the combination of the provided sound waves with angular frequencies ω_1 and ω_2 (where ω_2 is greater thanω_1) at the starting point or origin.

$$S_1 = A\sin\left(\omega_1 t + k_1 x\right)$$
$$S_2 = A\sin\left(\omega_2 t + k_2 x\right)$$



The phase difference between the waves is given by,

$$\Delta\phi = \left(\omega_2 - \omega_1\right)t$$

We know that for maxima to occur, $\Delta\phi$ should be an even multiple of π ($= 2n\pi$). Thus, for the first maxima,

$$\Delta\phi = 0$$
$$(\omega_2 - \omega_1)t_1 = 0$$
$$t_1 = 0$$

Similarly, for the second and the third maxima,

$$(\omega_2 - \omega_1)t_2 = 2\pi$$
$$t_2 = \frac{2\pi}{\omega_2 - \omega_1}$$
$$(\omega_2 - \omega_1)t_3 = 4\pi$$
$$t_3 = \frac{4\pi}{\omega_2 - \omega_1}$$

We can see that the time interval between maxima is constant and is given by

$$T_{\text{beats}} = \frac{2\pi}{\omega_2 - \omega_1} = \frac{1}{f_2 - f_1}$$

Hence, the beat frequency is given by,

$$f_{\text{beats}} = \frac{\omega_2 - \omega_1}{2\pi} = f_2 - f_1$$

A pattern of strong and weak sounds is called a beat. The rate at which these strong moments happen at a specific spot each second is called the beat frequency.

$$f_{\text{beats}} = |f_1 - f_2|$$

The inverse (or opposite) of the beat frequency is simply the amount of time it takes for two consecutive strong moments to occur.

For a beat frequency to be heard, $|f_1 - f_2|$ should not be very large. Practically, for beats to be heard, $f_{\text{beats}} < 10\ Hz$

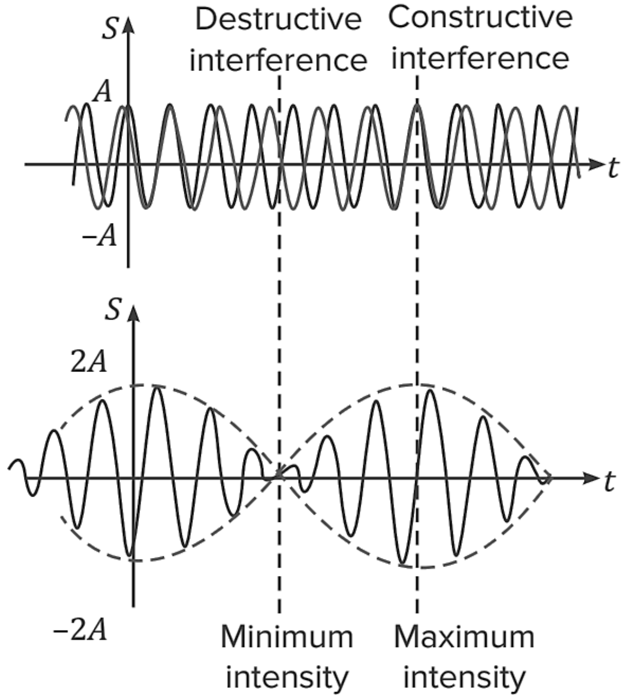
Beat Diagram

Now, we can find the total movement of the combined wave like this:

$$S = S_1 + S_2 = A\sin\omega_1 t + A\sin\omega_2 t$$
$$S = A(\sin\omega_1 t + \sin\omega_2 t)$$
$$S = 2A\sin\left(\frac{\omega_1 + \omega_2}{2}\right)t\cos\left(\frac{\omega_1 - \omega_2}{2}\right)t$$

$$S = 2A\sin 2\pi\left(\frac{f_1 + f_2}{2}\right)t\cos 2\pi\left(\frac{f_1 - f_2}{2}\right)t[\because \omega = 2\pi f]$$

Remember, for us to hear beats, the difference between the source frequencies must be small (less than 10 Hz). So, in the time it takes for one cycle of the cosine part, the sine part usually completes hundreds of cycles. That's why, for a brief moment, we can think of the combined wave as a simple up-and-down wave with a frequency of $\frac{f_1+f_2}{2}$. You can see how S_1 and S_2 come together in the picture provided.



Question.

A tuning fork A with a frequency of 500 Hz creates a pattern of five beats each second when it's used with another tuning fork B. If we modify tuning fork B through filing, the beat pattern changes to 8 Hz per second. Determine the frequency of tuning fork B.

Solution.

Given,

Frequency of tuning fork *A*, $f_A = 500\ Hz$

Beat frequency at the beginning, $f_{\text{beats}} = 5\ Hz$

If f_B is the frequency of tuning fork *B*, then we get,

$$|f_A - f_B| = 5$$

$$f_A - f_B = \pm 5$$

$$f_B = f_A \pm 5$$

$$f_B = 500 \pm 5$$

So, the potential frequencies for tuning fork B are 495 Hz and 505 Hz. According to the question, when we file the prongs of tuning fork B, its frequency goes up slightly. If the original frequency of tuning fork B is 495 Hz, filing it should raise it a bit, say to 497 Hz. This would make the beat frequency decrease to 3 Hz. However, in our situation, the beat frequency increases to 8 Hz. So, logically, the right frequency for tuning fork B should be 505 Hz to match the given conditions.

Question.

A tuning fork A with an original frequency of 500 Hz generates a pattern of eight beats each second when it's used alongside another tuning fork B. After applying wax to tuning fork A, the beat pattern shifts to 2 Hz. Determine the frequency of tuning fork B.

Solution.

Given,

Frequency of tuning fork *A*, $f_A = 500\ Hz$

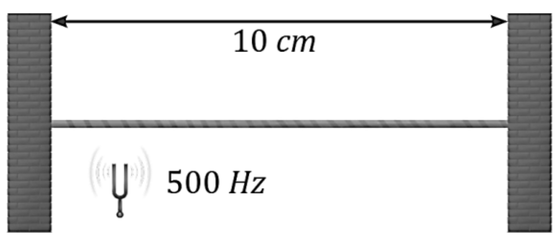
Beat frequency at the beginning, $f_{\text{beats}} = 8\ Hz$

Therefore, the frequency of tuning fork B (= fB) is (500 ± 8). It implies that fB is either 492 Hz or 508 Hz.

So, we put some wax on tuning fork A, which makes its frequency go down a bit. Let's say it's now a bit less than 500 Hz. If tuning fork B has a frequency of 508 Hz, the beat frequency increases. But in our case, the beat frequency goes down from 8 Hz to 2 Hz. So, it makes sense that the correct frequency for tuning fork B should be 492 Hz to match these conditions.

Question.

A string that's 10 cm long vibrates at its primary frequency when under 100 N of tension, as illustrated in the diagram. Determine the number of beat patterns it generates each second when a tuning fork with a frequency of 500 Hz is used. We also have information that the string has a mass density of 10 gm^{-1}.



Solution.

Given,

Length of the string, $L = 10\ cm = 0.10$ m

Linear mass density of the string, $\mu = 10gm^{-1} = 0.01\ kg\ \text{m}^{-1}$

Tension in the string, $T = 100\ N$

The fundamental frequency of the string is given by,

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

$$f = \frac{1}{2\times 0.1}\sqrt{\frac{100}{0.01}}$$

$$f = \frac{10}{0.2\times 0.1}$$

$$f = 500\ Hz$$

Because it's the same as the tuning fork's frequency, which is 500 Hz, when you play both of them together, there are no beat patterns. In other words, the beat frequency is zero.

Doppler Effect

When a person is on a train platform, they can hear a loud and high-pitched train horn as the train gets closer. But as the train moves away from the platform, the sound of the horn suddenly becomes lower in pitch and gradually gets quieter.

The Doppler Effect is when the pitch of a sound seems to change because the source of the sound and the listener are moving in relation to each other.

- The Doppler Effect is an occurrence perceived by an observer.
- The Doppler Effect becomes significant only when there is a dynamic change in the distance between the observer and the source. As a result, an observer positioned at the center of a circular railway track will not detect any fluctuation in the sound pitch of a train moving along that circular track.