FORCE AND LAWS OF MOTION

LAW OF CONSERVATION OF MOMENTUM

The momentum of an object is the product of the velocity and mass of an object. It is a vector quantity. Conservation of momentum is a fundamental law of physics, which states that the total momentum of an isolated system is conserved in the absence of an external force. In other words, the total momentum of a system of objects remains constant during any interaction if no external force acts on the system. The total momentum is the vector sum of individual momenta. Therefore, the component of the total momentum along any direction remains constant (whether the objects interact or not). Momentum remains conserved in any physical process.

Overview of the Law of Conservation of Momentum

Conservation of momentum states that the total momentum of an isolated system remains the same in the absence of an external force, i.e., the momentum can neither be created nor be destroyed, however, it can be changed through the action of forces as described by Newton's laws of motion.

Momentum is the product of the mass of the object and the velocity at which it is travelling and is also equal to the total force required to bring the object to rest.

One of the real-life aspects of the conservation of momentum is collision problems in which the momentum remains conserved and the net external force remains zero.

Additionally, there are several applications of momentum conservation in our day-to-day life that we will cover on this page. Along with this, we will understand the logic behind this concept and the proof of the conservation of momentum.

Illustration in One-Dimension

Conservation of momentum can be explained through a one-dimensional collision of two objects. Two objects of masses m1 and m2 collide with each other while moving along a straight line with velocities u1 and u2, respectively. After the collision, they acquire velocities v_1 and v_2 in the same direction.

Total momentum before collision $p_i=m_1u_1+m_2u_2$

Total momentum after collision $pf=m_1v_1+m_2v_2$

If no other force acts on the system of the two objects, total momentum remains conserved. Therefore,

$P_i = p_f$

$m_1u_1+m_2u_2= m_1v_1+m_2v_2$

Derivation of Conservation of Momentum

If no external force is exerted on the system of two colliding objects, the objects apply impulse on each other for a short interval of time at the point of contact. According to Newton's third law of motion, the impulsive force applied by the first object on the second one is equal and opposite to the impulsive force applied by the second object on the first object.

During the one-dimensional collision of two objects of masses m_1 and m_2, which have velocities u_1 and u_2 before collision and velocities v_2 and v_2 after the collision, the impulsive force on the first object is F_{21} (applied by the second object) and the impulsive force on the second object is F_{12} (applied by the first object). Applying Newton's third law, these two impulsive forces are equal and opposite, i.e.,

$F_{21} = - F_{12}$

If the time of contact is t, the impulse of the force F_{21} is equal to the change in momentum of the first object.

$F_{21}. t = m_1v_1 - m_1u_1$

The impulse of force F_{12} is equal to the change in momentum of the second object.

$F_{12}\,t = m_2v_2 - m_2u_2$

From $F_{21} = - F_{12}$

$F_{21}.\,t = - F_{12}.\,t$

$m_1v_1 - m_1u_1 = - (m_2v_2 - m_2u_2)$

$m_1u_1+m_2u_2= m_1v_1+m_2v_2$

This relation suggests that momentum is conserved during the collision.

Collision in Two – Dimensions

Before the collision, the total momentum is pix = p1 = m1v1, along the X - axis and piy = p2 = m2v2 along the Y - axis. After the collision, the total momentum is

$p_{fx} = (m + M)$ ucosθ, along X-axis and $p_{fy} = (m+M)$ usinθ

Applying conservation of momentum,

$p_{ix} = p_{fx}$

$m_1v_1 = (m + M)\,u\cos\theta$ …. (1)

$p_{iy} = p_{fy}$

$m_2v_2 = (m+M)\,u\sin\theta$ …. (2)

Therefore, squaring and adding equations (1) and (2),

$(m_1v_1)^2+(m_2v_2)^2=(m+M)^2u^2(Co^{s2}\Theta+Sin^2\Theta)$

$$u = \frac{\sqrt{m_1^2v_1^2 + m_2^2v_2^2}}{(m+M)}$$

It is the speed of the combined object.

Dividing equation (2) by (1),

$$t\,an\,\Theta = \frac{m_2v_2}{m_1v_1}$$

θ gives the direction of the velocity.

Conservation of Momentum Examples

Recoil of a Gun:

If a bullet is fired from a gun, both the bullet and the gun are initially at rest i.e. the total momentum before firing is zero. The bullet acquires a forward momentum when it gets fired. According to the conservation of momentum, the gun receives a backward momentum. The bullet of mass m is fired with forward velocity v. The gun of mass M acquires a backward velocity u. Before firing, the total momentum is zero so that the total momentum after firing is also zero.

$0 = mv + Mu$

$$u = -\frac{m}{M}V$$

u is the recoil velocity of the gun. The mass of the bullet is much less than that of the gun i.e. $m \ll M$. The backward velocity of the gun is very small,

$u \ll v$

Rocket Propulsion:

Rockets have a gas chamber at one end, from which gas is ejected with enormous velocity. Before the ejection, the total momentum is zero. Due to the ejection of gas, the rocket gains a recoil velocity and acceleration in the opposite direction. This is a consequence of the conservation of momentum

If a rocket of mass m ejects the propellent of small mass dm with an exhaust velocity ve such that the residual rocket of mass m - dm acquires a velocity dv in the opposite direction, the momenta of the propellant and the residual rocket are equal in magnitude and opposite in direction.

$v_e\, dm = -\,(m - dm)\, dv$

Since both dm and dv are small, the equation can be approximated as

$$dv = -v_e \frac{dm}{m}$$

If the mass of the rocket reduces from mo to m' as its velocity increases from 0 to v', integrating the above equation

$$V' = V_e in\left(\frac{m}{mo}\right)$$

Solved Examples

A bullet of mass 6 g is fired with a speed of 500 m/s from a gun of mass 4 kg. What would be the recoil velocity of the gun?

Solution: The initial momenta of the bullet and the gun are zero such that the total initial momentum is zero. The bullet of mass m = 6g is fired with forward velocity v = 500 m/s. The gun of mass M = 4kg acquires a backward velocity V.

$$m = 6g = \frac{6}{1000}kg$$

According to the conservation of momentum formula,

0 = mv + MV

$$0 = \frac{6}{1000}kg(500m/s) + (4kg)V$$

v = - 0.75 m/s

The recoil speed of the gun is 0.75 m/s.

The negative sign implies that the recoil velocity is opposite to the velocity of the bullet.