MOTION

CIRCULAR MOTION

Circular Motion

Circular motion is described as a movement of an object while rotating along a circular path. Circular motion can be either uniform or non-uniform. During uniform circular motion the angular rate of rotation and speed will be constant while during non-uniform motion the rate of rotation keeps changing.

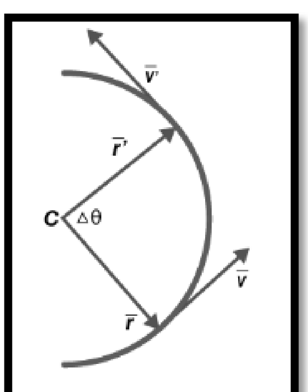
Some of the most common examples of circular motion include man-made satellite that revolves around the earth, a rotating ceiling fan, a moving car's wheel, the blades in a windmill, and gears in gas turbines.

A particle is said to execute circular motion when it moves along the circumference of a circular path. An important aspect of circular motion is that the direction of motion is changing continuously unlike in the case of linear motion. Hence circular motion can also be described in terms of angular variables.

Angular Variables

Angular Displacement

It is defined as the angle turned by a rotating particle per unit time. It is represented by $\Delta\theta$ and measured in radians. In the figure, angular displacement is measured between the position vectors rand r'.



Angular Velocity

It is defined as the rate of change in angular displacement of a particle in a circular motion. It is denoted by

$$\omega = \lim_{\Delta t \to 0}\left(\Delta\theta / \Delta t\right) = d\theta / dt$$

Angular velocity is measured in rad/s. Apart from angular velocity and angular speed, a particle in circular motion also possesses linear velocity and corresponding linear speed.

$$v = ds / dt$$

$v = \left|ds / dt\right|; s$ is the displacement of the particle

Relation Between Linear Speed(V) And Angular Speed(Ω)

In vector form

$v = \omega \times r$

Where r is the position vector of the particle measured with respect to the centre of the circle.

(Or)

$v = r\omega$

The acceleration of a particle in circular motion has two components:

Tangential acceleration at: This is the component of acceleration in the direction of the velocity of the particle.

$a_t = d|v|/dt$

Radial acceleration ar:

This is the component of acceleration directed towards the centre of the circle. This component causes a change in the direction of the velocity of a particle in a circular motion.

$a_r = v^2/r = r\omega^2$

Angular Acceleration

It is defined as the rate of change of angular velocity of the rotating particle. It is measured in rad/s^2

$\alpha = d\omega/dt = d^2\theta/dt^2$

Circular motion can be uniform and non-uniform depending on the nature of acceleration of the particle. The motion is called uniform circular motion when the particle is moving along a circular path possessing a constant speed.

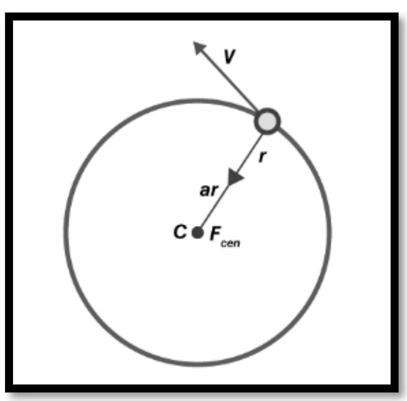
During circular motion, the velocity vector changes its direction at each point on the circle. This implies that the radial component of acceleration is always non-zero. The tangential component can take a positive or negative value in the case of non-uniform circular motion and a zero value in the case of uniform circular motion.

We understand that the acceleration of a particle in a circular motion is always directed towards the center and is given by v^2/r. Applying Newton's second law of motion in this situation;

$F_{cen} = mv^2/r$

Where m is the mass of the particle

F_c is the centripetal force directed towards the centre of the circular path as illustrated in



Examples of centripetal forces include gravitational force, the tension in a string and friction.

Application of Newton's Laws of Motion

Let us understand the application of Newton's laws of motion in two different scenarios. The motion of a vehicle will be on either:

At Level Road

At an instant when a vehicle makes a turn on a level road, there are three different forces acting.

The weight of the car W= mg

Frictional force f_f

Normal reaction N

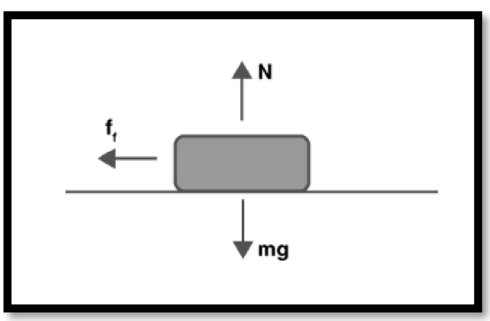
In the vertical direction N-mg = 0

N = mg

The centripetal acceleration for circular motion is provided by static friction. This friction holds back the vehicle from moving away from the circle.

$f_f = mv^2/r \leq \mu N$

μis the coefficient of static friction

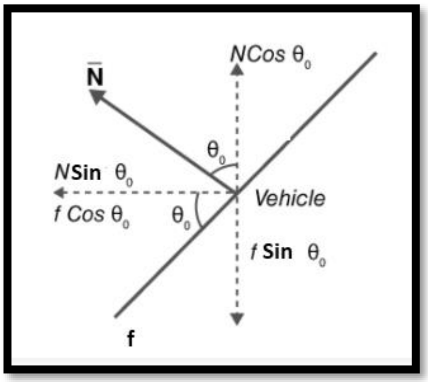


$v_{max} = \sqrt{\mu r g}$

This implies that for a circular path of given radius and μ, there is a maximum possible speed of the vehicle.

At Banked Roads

The effect of friction on the motion of a vehicle on a circular road can be minimized if the road is slightly raised on the outer end. This is called banking. Let the road be banked at an angle θ0 as illustrated in the figure.



Net force along the vertical direction is zero since there is no acceleration along this direction.

Therefore,

$N \cos \theta_0 = f \sin \theta_0 + mg$ ..... (1)

The centripetal force is provided by the horizontal component of N and f

$mv^2/r = N \sin \theta_0 + f \cos \theta_0$ .... (2)

Substituting f= μN in equations (1) and (2) gives v_{max}

$N \cos \theta_0 = \mu N \sin \theta_0 + mg$

$mv^2/r = N \sin \theta_0 + \mu N \cos \theta_0$

Solving the above equations gives us,

$v_{max} = \sqrt{rh \tan \theta_0}$

$\tan \theta_0 = v_{max}^2/rg$