GRAVITATION

KEPLER'S LAW

Introduction to Kepler's Laws

Motion is always relative. Based on the energy of the particle under motion, the motions are classified into two types:

Bounded Motion

Unbounded Motion

In bounded motion, the particle has negative total energy ($E < 0$) and has two or more extreme points where the total energy is always equal to the potential energy of the particle, i.e., the kinetic energy of the particle becomes zero.

For eccentricity $0 \leq e < 1$, $E < 0$ implies the body has bounded motion. A circular orbit has eccentricity $e = 0$, and an elliptical orbit has eccentricity $e < 1$.

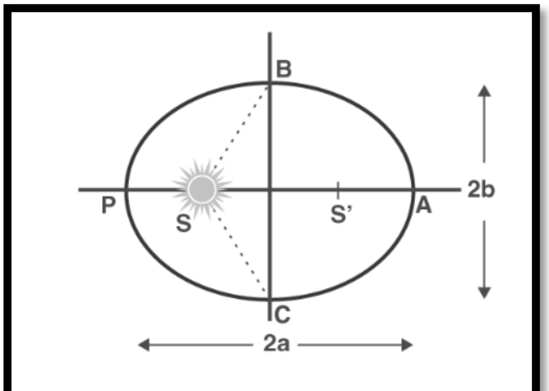
In unbounded motion, the particle has positive total energy ($E > 0$) and has a single extreme point where the total energy is always equal to the potential energy of the particle, i.e., the kinetic energy of the particle becomes zero.

For eccentricity $e \geq 1$, $E > 0$ implies the body has unbounded motion. Parabolic orbit has eccentricity $e = 1$, and Hyperbolic path has eccentricity $e > 1$.

Kepler First law – The Law of Orbits

According to Kepler's first law," All the planets revolve around the sun in elliptical orbits having the sun at one of the foci". The point at which the planet is close to the sun is known as perihelion, and the point at which the planet is farther from the sun is known as aphelion.

It is the characteristic of an ellipse that the sum of the distances of any planet from two foci is constant. The elliptical orbit of a planet is responsible for the occurrence of seasons.



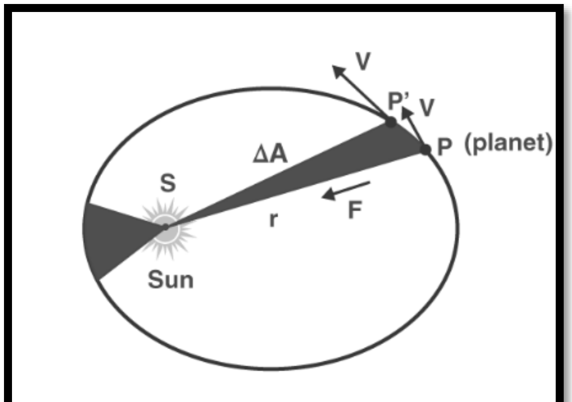
Kepler First Law – The Law of Orbits

Kepler's Second Law – The Law of Equal Areas

Kepler's second law states," The radius vector drawn from the sun to the planet sweeps out equal areas in equal intervals of time"

As the orbit is not circular, the planet's kinetic energy is not constant in its path. It has more kinetic energy near the perihelion, and less kinetic energy near the aphelion implies more speed at the perihelion and less speed (v_{min}) at the aphelion. If r is the distance of planet from sun, at perihelion (r_{min}) and at aphelion (r_{max}), then,

$r_{min} + r_{max} = 2a \times$ (length of major axis of an ellipse) ... (1)



Kepler's Second Law – The law of Equal Areas

Using the law of conservation of angular momentum, the law can be verified. At any point of time, the angular momentum can be given as, $L = mr2\omega$.

Now consider a small area ΔA described in a small time interval Δt and the covered angle is $\Delta\theta$. Let the radius of curvature of the path be r, then the length of the arc covered = $r\,\Delta\theta$.

$\Delta A = 1/2[r.(r.\Delta\theta)] = 1/2r^2\Delta\theta$

Therefore, $\Delta A/\Delta t = [\,1/2r^2]\Delta\theta/dt$

Taking limits on both sides as, $\Delta t \rightarrow 0$, we get;

$$\lim_{\Delta t\to 0}\frac{\Delta A}{\Delta t} = \lim_{\Delta t\to 0}\frac{1}{2}r^2\frac{\Delta\theta}{\Delta t}$$

$$\frac{dA}{dt} = \frac{1}{2}r^2\omega$$

$$\frac{dA}{dt} = \frac{L}{2m}$$

Now, by conservation of angular momentum, L is a constant

Thus, dA/dt = constant

The area swept in equal intervals of time is a constant.

Kepler's second law can also be stated as "The areal velocity of a planet revolving around the sun in elliptical orbit remains constant, which implies the angular momentum of a planet remains constant". As the angular momentum is constant, all planetary motions are planar motions, which is a direct consequence of central force.

Kepler's Third Law – The Law of Periods

According to Kepler's law of periods," The square of the time period of revolution of a planet around the sun in an elliptical orbit is directly proportional to the cube of its semi-major axis".

$$T^2 \alpha a^3$$

Shorter the orbit of the planet around the sun, the shorter the time taken to complete one revolution. Using the equations of Newton's law of gravitation and laws of motion, Kepler's third law takes a more general form:

$$P^2 = 4\pi^2 /[G(M_1+ M_2)] \times a^3$$

where M_1 and M_2 are the masses of the two orbiting objects in solar masses.