AREAS OF PARALLELOGRAMS AND TRIANGLES

AREA OF TRIANGLE AND PARALLELOGRAM BETWEEN THE SAME BASE AND SAME PARALLEL LINE

AREA OF A TRIANGLE

Theorem-5 :

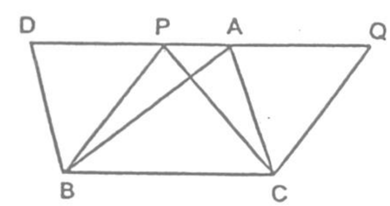
Two triangles on the same base (or equal bases) and between the same parallels are equal in area.

Given : Two triangles ABC and PCs on the same base BC and between the same parallel lines BC and AP.

To prove : ar(ΔABC) = ar(ΔPBC)

Construction : Through B, draw BD ∥ CA intersecting PA produced in D and through C, draw CQ ∥ BP, intersecting line AP in Q.

Proof :



We have,

BD ∥ CA [By construction]

And, BC ∥ DA [Given]

Quad. BCAD is a parallelogram.

Similarly, Quad. BCQP is a parallelogram.

Now, parallelogram BCQP and BCAD are on the same base BC, and between the same parallels.

ar($\|^{gm}$BCQP) = ar($\|^{gm}$BCAD) ....(i)

We know that the diagonals of a parallelogram divides it into two triangles of equal area.

$\therefore$ ar(ΔPBC$=\frac{1}{2}$ar($\|^{gm}$BCQP) ....(ii)

And ar(ΔABC) $=\frac{1}{2}$ar($\|^{gm}$BCAD) ....(iii)

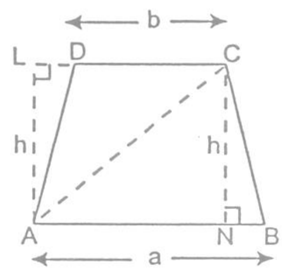
Now, ar($\|^{gm}$ BCQP) = ar($\|^{gm}$BCAD) [From (i)]

$\Rightarrow$ $\frac{1}{2}$ar($\|^{gm}$BCAD) $=\frac{1}{2}$ar($\|^{gm}$BCQP)

Hence, ar(ΔABC) = ar(ΔPBC) [Using (ii) and (iii)] **Hence Proved.**

Theorem-6 :

The area of a trapezium is half the product of its height and the sum of the parallel sides.



Given : Trapezium ABCD in which AB $\|$ DC, AL $\perp$ DC, CN $\perp$AB and AL = CN = h (say)

AB = a, DC = b.

To prove : ar(trap. ABCD) $=\frac{1}{2}$h $\times$ (a + b).

Construction : Join AC.

Proof :

AC is a diagonal of quad. ABCD.

ar(trap. ABCD) = ar(ΔABC) + ar(ΔACD) $=\frac{1}{2}$h $\times$ a $+\frac{1}{2}$h $\times$ b$=\frac{1}{2}$h(a + b).

Hence Proved.

Theorem -7:

Triangles having equal areas and having one side of the triangle equal to corresponding side of the other, have their corresponding altitudes equal

Given : Two triangles ABC and PQR such that

(i) ar (ΔABC) = ar(ΔPQR) and

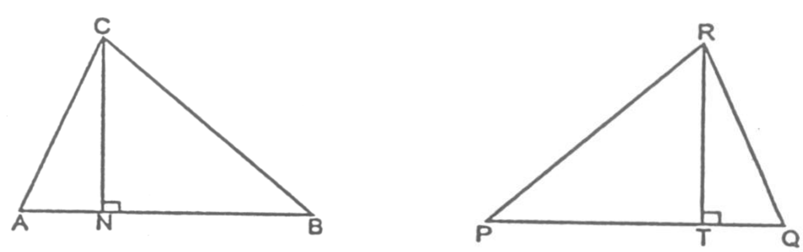
(ii) AB = PQ.

CN and RT and the altitude corresponding to AB and PQ respectively of the two triangles.

To prove : CR = RT.

Proof : In ΔABC, CN is the altitude corresponding to the side AB.

$$\text{ar}(\Delta ABC) = \frac{1}{2} AB \times CN \quad \text{....(i)}$$



Similarly, $\text{ar}(\Delta PQR) = \frac{1}{2} PQ \times RT$ ....(ii)

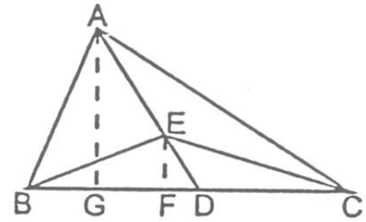
Since $\text{ar}(\Delta ABC) = \text{ar}(\Delta PQR)$ [Given]

$\therefore$ $\frac{1}{2} AB \times CN = \frac{1}{2} PQ \times RT$

Also, $AB = PQ$ [Given]

$CN = RT$ **Hence Proved.**

Ex.1 In figure, E is any point on median AD of a ΔABC. Show that ar(ABE) = ar(ACE).



Sol. **Construction :** From A draw AG $\perp$ BC and from E draw EF $\perp$ BC.

Proof :

ar(ΔABD) = $\frac{BD \times AG}{2}$

ar(ΔADC) = $\frac{DC \times G}{2}$

But, BD = DC [$\therefore$D is the mid-point of BC, AD being the median]

ar(ΔABD) = ar(ΔADC) ...(i)

Again, ar(ΔEBD)= $\frac{BD \times EF}{2}$

ar(ΔEDC) = $\frac{DC \times EF}{2}$

But, BD = DC

$\therefore$ ar(ΔEBD) = ar(ΔEDC) ...(ii)

Subtracting (ii) from (i), we get

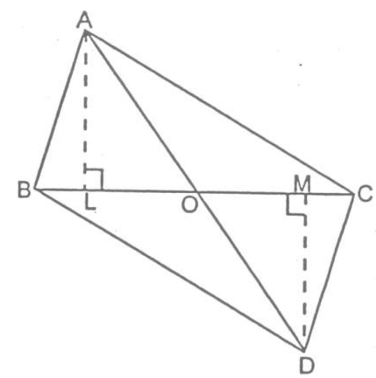
ar(ΔABD) - ar(ΔEBD) = ar(ΔADC) - ar(ΔEDC)

$\Rightarrow$ ar(ΔABE) = ar(ΔACE). **Hence Proved.**

Ex.2 Triangles ABC and DBC are on the same base BC; with A, D on opposite sides of the line BC, such that ar(ΔABC) = ar(ΔDBC). Show that BC bisects AD.

Sol. **Construction :** Draw AL $\perp$ BC and DM $\perp$ BC.

Proof :



ar(ΔABC) = ar(ΔDBC) [Given]

$\Rightarrow \frac{BC \times AL}{2} = \frac{BC \times DM}{2}$

$\Rightarrow$ AL = DM ....(i)

Now in Δs OAL and OMD

AL = DM [From (i)]

$\Rightarrow$ $\angle$ALO = $\angle$DMO [Each = 90^0]

$\Rightarrow$ $\angle$AOL = $\angle$MOD [Vert. opp. $\angle$s]

$\Rightarrow$ $\angle$OAL = $\angle$ODM [Third angles of the triangles]

$\therefore$ $\Delta OAL \cong \Delta OMD$ [By ASA]

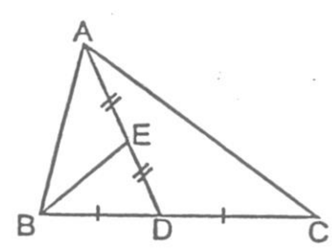
$\therefore$ OA = OD [By cpctc]

i.e., BC bisects AD. **Hence Proved.**

Ex.3 ABC is a triangle in which D is the mid-point of BC and E is the mid-point of AD. Prove that the area of ΔBED$=\frac{1}{4}$area of ΔABC.

Sol. **Given :** A ΔABC in which D is the mid-point of BC and E is the mid-point of AD.

To prove: ar(ΔBED) = $\frac{1}{4}$ar(ΔABC).



Proof :

$\because$AD is a median of ΔABC.

ar(ΔABD) = ar(ΔADC) = $\frac{1}{2}$ar(ΔABC) .....(i)

[$\therefore$Median of a triangle divides it into two triangles of equal area) $= \frac{1}{2}$ar(ΔABC)

Again,

$\because$ BE is a median of ΔABD,

$\therefore$ ar(ΔBEA) = ar(ΔBED) = $\frac{1}{2}$ar(ΔABD)

[$\therefore$Median of a triangle divides it into two triangles of equal area]

And $\frac{1}{2}$ar(ΔABD) = $\frac{1}{2}\times\frac{1}{2}$ar($\Delta$ABC) [From (i)]

$\therefore$ ar(ΔBED) = $\frac{1}{4}$ar(ΔABC). **Hence Proved.**

Ex.4 if the medians of a ΔABC intersect at G, show that ar(ΔAGB) = ar(ΔBGC) = $\frac{1}{3}$ ar(ΔABC).

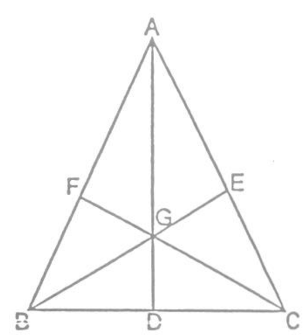
Sol. **Given :** A ΔABC its medians AD, BE and CF intersect at G.

To prove : ar(ΔAGB) = ar(ΔAGC) = ar(ΔBGC) = $\frac{1}{3}$ar(ΔABC).

Proof :

A median of triangle divides it into two triangles of equal area.

In ΔABC, AD is the median.



$\therefore$ ar(ΔABD) = ar(ΔACD) ...(i)

In ΔGBC, GD is the median.

$\therefore$ ar(ΔGBD) = ar(ΔGCD) ...(ii)

From (i) and (ii), we get

ar(ΔABD) - ar(ΔGBD) = ar(ΔACD) -ar(ΔGCD)

$\therefore$ a(ΔAGB) = ar(ΔAGC).

Similarly,

ar(ΔAGB) =ar(ΔAGC) =ar(ΔBGC) ....(iii)

But, ar(ABC) = ar(ΔAGB) + ar(ΔAGC) + ar(ΔBGC)

= 3 ar(ΔAGB) [Using (iii)]

$\therefore$ ar(ΔAGB) = $\frac{1}{3}$ ar(ΔABC).

Hence, ar(ΔAGB) = ar(ΔAGC) = arΔ(BGC) = $\frac{1}{3}$ ar(ΔABC). **Hence proved.**