AREAS OF PARALLELOGRAMS AND TRIANGLES

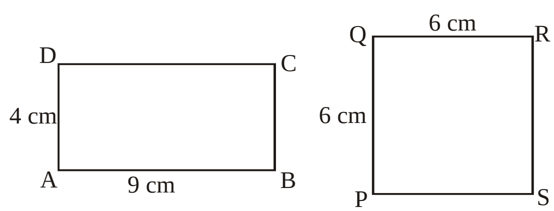
AREA OF PARALLELOGRAM BETWEEN SAME BASE AND SAME PARALLEL LINE

INTRODUCTION

The part of the plane enclosed by a simple closed figure is called a planar region corresponding to that figure. The magnitude or measusre of this planar region is called its area. In this chapter, you will learn to find the area of different plane figures, such as a triangle, rectangle, parallelogram etc by studying some relationship between the areas of these figures under the condition when they lie on the same base and between the same parallels lines.

CONCEPT OF CONGRUENT FIGURES WITH RESPECT TO AREA

Two figures are called congruent, if they have the same shape and the same size. In other words if two figures A and B are congruent then one figure can be superimposed over the other such that it will cover the other completely. So if two figures A and B are congurent, they must have equal areas. However the converse of this statement is not true i.e. if two figures having equal areas then they need not be congruents. For example in figure rectangles ABCD & PQRS have equal areas (9×4 cm^2 & 6×6 cm^2 but clearly they are not congruent.



1. Area Axioms

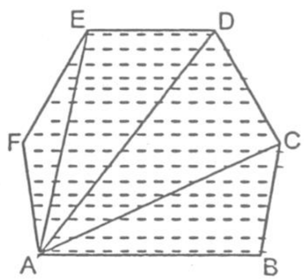
(a) Every planar region R has an area measured in square units and denoted by ar (R).

(b) If A and B are two congruent figures then ar(A) = ar(B).

(c) If a planar region R is made up of two non-overlapping planar regions P and Q then ar(R) = ar(P) + ar(Q)

POLYGONAL REGION

Polygon region can be expressed as the union of a finite number of triangular regions in a plane such that if two of these intersect, their intersection is either a point or a line segment. It is the shaded portion including its sides as shown in the figure.



(a) Area Axioms :

Every polygonal region R has an area, measure in square unit and denoted by ar(R).

(i) **Congruent area axiom :** if R_1 and R_2 be two regions such that $R_1 \cong R_2$ then $ar(R_1) = ar(R_2)$.

(ii) **Area monotone axiom :** If $R_1 \subset R_2$, then are $(R_1) \le ar(R_2)$.

(iii) **Area addition axiom :** If R_1 are two polygonal regions, whose intersection is a finite number of points and line segments and $R = R_1 \cup R_2$, then $ar(R) = ar(R_1) + ar(R_2)$.

(iv) **Rectangular area axiom :** If AB = a metre and AD = b metre then,

ar (Rectangular region ABCD) = ab sq. m.

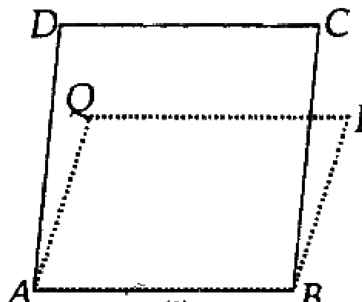
(b) Unit of Area :

There is a standard square region of side 1 metre, called a square metre, which is the unit of area measure. The area of a polygonal region is square metres (sq. m or m^2) is a positive real number

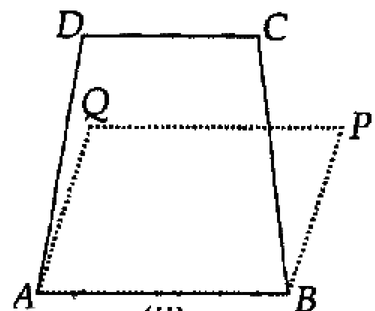
FIGURES ON THE SAME BASE AND BETWEEN THE SAME PARALLES

Look at the following figures (i) parallelogram ABCD and ABPQ on the same base AB and in fig. (ii), trapezium ABCD and parallelogram ABPQ are on the same base AB. If two geometric figures have a common side, we say that they are on the same base.

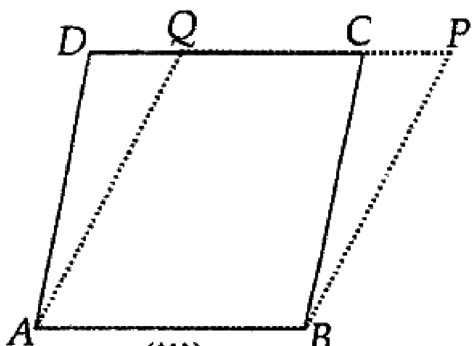
In fig. (iii) parallelograms ABCD and ABPQ are on the same base AB and between the same paralles AB and DP as the vertices C & D of parallelogram ABCD and P & Q of parallelogram ABPQ lie on a line DP parallel to base AB. In the same way, we can say that in fig. (iv) ΔABP and parallelogram ABCD are on the same base AB and between the same parallel lines AB and DP.



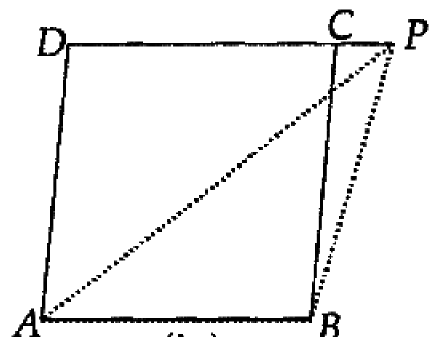
(i)



(ii)



(iii)



(iv)

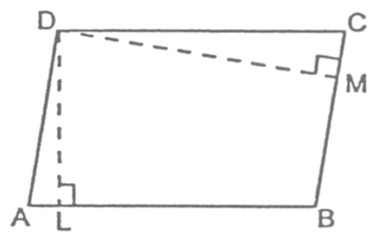
Thus, two geometric figures are said to be on the same base and between the same parallel lines, if they have a common side (base) and the vertices (or the vertex) opposite to the common base of each figure lie on a line parallel to the base.

AREA OF A PARALLELOGRAM

(a) Base and Altitude of a Parallelogram :

(i) Base : Any side of parallelogram can be called its base.

(ii) Altitude : The length of the line segment which is perpendicular to the base from the opposite side is called the altitude or height of the parallelogram corresponding to the given base.



In the Adjoining Figure

(i) DL is the altitude of $\|^{gm}$ ABCD, corresponding to the base AB.

(ii) DM is the altitude of $\|^{gm}$ ABCD, corresponding to the base BC.

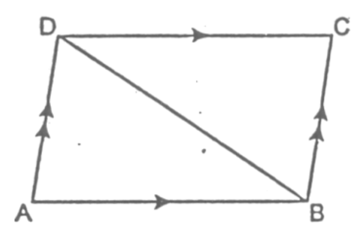
THEOREM 1.

A diagonal of parallelogram divides it into two triangles of equal area.

Given : A parallelogram ABCD whose one of the diagonals is BD.

To prove : ar (ΔABD) = ar (ΔCDB).

Proof : In ΔABD and ΔCDB.

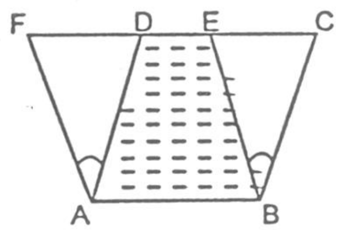


AB = DC	[Opp. sides of a $\|^{gm}$]
AD = BC	[Opp. sides of a $\|^{gm}$]
BD = BD	[Common side]
$\therefore$ $\Delta ABD \cong \Delta CDB$	[By SSS]
$\therefore$ ar (ΔABD) = ar(ΔCDB)	[Congruent area axiom]

Hence Proved.

Theorem 2:

Parallelograms on the same base or equal base and between the same parallels are equal in area.



Given : Two $\|^{gm}$ ABCD and ABEF on the same base AB and between the same parallels AB and FC.

To Prove : $ar(\|^{gm} ABCD) = ar(\|^{gm} ABEF)$

Proof : In ΔADF and ΔBCE, we have

$AD = BC$ [Opposite sides of a $\|^{gm}$]

$AF = BE$ [Opposite sides of a $\|^{gm}$]

$\angle DAF = \angle CBE$ [$\because AD \parallel BC$ and $AF \parallel BE$]

[Angle between AD and AF = angle between BC and BE]

$\therefore \Delta ADF \cong \Delta BCE$ [By SAS]

$\therefore ar(\Delta ADF) = ar(\Delta BCE)$ ....(i)

$\therefore ar(\|^{gm} ABCD) = ar(ABED) + ar(\Delta BCE)$

$= ar(ABED) + ar(\Delta ADF)$ [Using (i)]

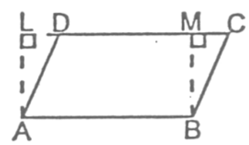
$= ar(\|^{gm} ABEF)$.

Hence, $ar(\|^{gm} ABCD) = ar(\|^{gm} ABEF)$. **Hence Proved.**

NOTE : A rectangle is also parallelogram.

Theorem 3:

The are of parallelogram is the product of its base and the corresponding altitude.



Given : A $\|^{gm}$ ABCD in which AB is the base and AL is the corresponding height.

To prove : Area ($\|^{gm}$ABCD) = AB × AL.

Construction : Draw BM $\perp$ DC so that rectangle ABML is formed.

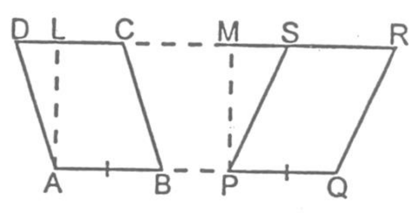
Proof : $\|^{gm}$ ABCD and rectangle ABML are on the same base AB and between the same parallel lines AB and LC.

$\therefore$ ar($\|^{gm}$ABCD) = ar(rectangle ABML) = AB × AL.

$\therefore$ area of a $\|^{gm}$ = base × height. **Hence Proved.**

Theorem 4 :

Parallelograms on equal bases and between the same parallels are equal in area.



Given : Two $\|^{gm}$ ABCD and PQRS with equal base AB and PQ and between the same parallels, AQ and DR.

To prove: ar($\|^{gm}$ ABCD) = ar($\|^{gm}$PQRS).

Construction : Draw AL $\perp$ DR and PM $\perp$DR.

Proof : AB $\parallel$ DR, AL$\perp$DR and PM$\perp$Dr

$\therefore$ AL = PM.

$\therefore$ ar($\parallel^{gm}$ ABCD) = AB × AL

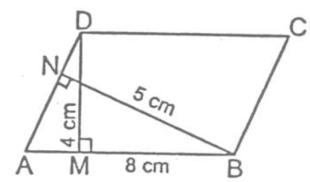
= PQ × PM [$\because$AB = PQ and AL = PM]

= a($\parallel^{gm}$PQRS). **Hence Proved.**

Ex.1 In a parallelogram ABCD, AB = 8 cm. The altitudes corresponding to sides AB and AD are respectively 4 m and 5 cm. Find AD.

Sol. We know that, Area of a parallelogram = Base × Corresponding altitude

Area of parallelogram ABCD = AD × BN = AB × DM

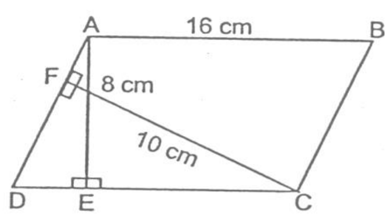


$\Rightarrow$ AD × 5 = 8 × 4

$\Rightarrow$ AD = $\frac{8 \times 4}{5}$

= 6.4 cm. **Ans.**

Ex.2 In figure, ABCD is a parallelogram, AE $\perp$ DC and CF$\perp$AD. If AB = 16 cm, AE = 8 cm and CF = 10 cm find AD.



Sol. We have AB = 16 cm, AE = 8 cm CF = 10 cm.

We know that are of parallelogram = Base × Height [Base = CD, height = AE]

ABCD = CD × AE = 16 × 8 = 128 cm^2

Again, Area of parallelogram = Base × Height = AD × CF [Base = AD, height = CF]

128 = AD × 10

AD = $\frac{128}{10}$=12.8 cm **Ans.**

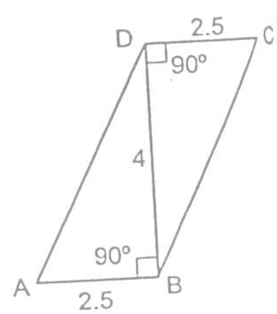
Ex.3 ABCD is a quadrilateral and BD is one of its diagonal as shown in the figure. Show that the quadrilateral ABCD is a parallelogram and find its area.

Sol. From figure, the transversal DB is intersecting a pair of lines DC and AB such that

$\angle CDB = \angle ABD = 90^0$.

Hence these angles from a pair of alternate equal angles.

∴ DC ∥ AB.



Also DC = AB = 2.5 units.

∴ Quadrilateral ABCD is a parallelogram.

Now, area of parallelogram ABCD

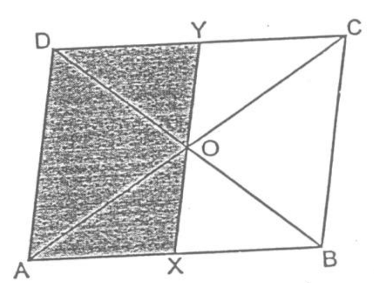
= Base × Corresponding altitude

= 2.5 × 4

= 10 sq. units **Ans.**

Ex.4 The diagonals of a parallelogram ABCD intersect in O. A line through O meets AB is X and the opposite side CD in Y. Show that ar (quadrilateral AXYD) = $\frac{1}{2}$far(parallelogram ABCD).

Sol. ∴ AC is a diagonal of the parallelogram ABCD.



ar(ΔACD) = $\frac{1}{2}$ar(ABCD) ...(i)

Now, in Δs AOX and COY,

AO = CO

$\because$ Diagonals of parallelogram bisect each other.

$\angle AOX = \angle COY$ [Vert. opp. $\angle$s]

$\angle OAX = \angle OCY$ [Alt. Int. $\angle$s]

$\therefore$ AB $\parallel$ DC and transversal AC intersects them

$\therefore$ $\Delta AOX \cong \Delta COY$ [ASA]

$\therefore$ $ar(\Delta AOX) = ar(\Delta COY)$ ....(ii)

Adding ar(quad. AOYD) to both sides of (ii), we get

$ar(\text{quad. } AOYD) + ar(\Delta AOX) = ar(\text{quad. } AOYD) + ar(\Delta COY)$

$\Rightarrow$ $ar(\text{quad. } AXYD) = ar(\Delta ACD) = \frac{1}{2} ar(\parallel gm\ ABCD)$ (using (i)) **Hence Proved.**