QUADRILATERALS

BASIC CONCEPT OF QUADRILATERALS

INTRODUCTION

A quadrilateral is a closed figure obtained by joining four points in a plane (with no three points collinear) in an order.

Since, 'quad' means 'four' and 'lateral' means 'sides' , therefore 'quadrilateral' means 'a figure bounded by four sides'.

Every quadrilateral has :

(i) Four vertices

(ii) Four sides

(iii) Four angles and

(iv) Two diagonals.

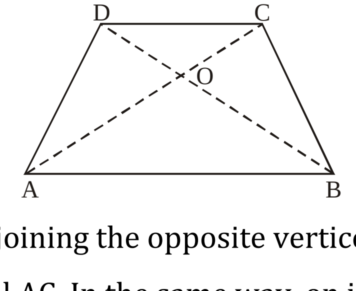
The given figure shows a quadrilateral ABCD, which has

(i) four vertices namely : A, B, C and D.

(ii) four sides namely : AB, BC , CD and DA

(iii) four angles namely : $\angle A$, $\angle B$, $\angle C$ and $\angle D$

(iv) two diagonals, namely : AC and BD



A diagonal is a line segment obtained on joining the opposite vertices. Thus on joining opposite vertices A and C, we get diagonal AC. In the same way, on joining the opposite vertices B and D, we get the diagonal BD.

Adjacent sides :

Two sides of a quadrilateral having a common end point are called its adjacent or consecutive sides. (AB, BC), (BC, CD), (CD, DA) and (DA, AB) are four pairs of its adjacent sides.

Opposite sides :

Two sides of a quadrilateral having no common end point are called its opposite sides.

Adjacent Angles :

Two angles of a quadrilateral having a common arm are called its adjacent angles.

($\angle A$, $\angle B$), ($\angle B$, $\angle C$), ($\angle C$, $\angle D$) and ($\angle D$, $\angle A$) are four pairs of adjacent angles.

Opposite angles :

Two angles of a quadrilateral having no common arm are called its opposite angles.

($\angle A$, $\angle C$) and ($\angle B$, $\angle D$) are two pairs of opposite angles.

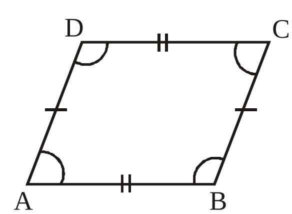
VARIOUS TYPES OF QUADRILATERALS

1 Parallelogram

In a quadrilateral, if both the pairs of opposite sides are parallel, then it is called a parallelogram.

Properties of parallelogram:

In the given figure,



(i) AB = CD and BC = AD

(ii) and . Hence, ABCD is a parallelogram.

(iii) Each pair of opposite angles are equal,

i.e. $\angle A = \angle C$ and $\angle B = \angle D$.

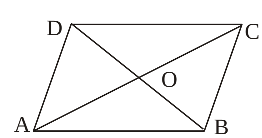
(iv) The diagonals AC and BD bisect each other i.e. AO = OC and BO = OD.

(v) Sum of each pair of adjacent angles is 180°, i.e.

$\angle A + \angle B = \angle B + \angle C = \angle C + \angle D = \angle D + \angle A = 180°$.

(vi) Each diagonal divides the parallelogram into two congruent triangles.

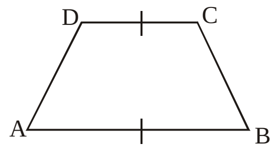
i.e. $\Delta ABC \cong \Delta CDA$ and $\Delta ABD \cong \Delta CDB$.



Note: In a parallelogram, diagonals need not be equal, but they bisect each other.

2 Trapezium

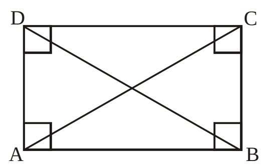
In a quadrilateral, if two opposite sides are parallel to each other, then it is called a trapezium.



In the given figure $\overline{AB} \parallel \overline{CD}$, hence ABCD is a trapezium.

3 Rectangle

In a parallelogram, if each angle is a right angle (90º), then it is called a rectangle.



In the given figure, $\angle A = \angle B = \angle C = \angle D = 90º$, AB = CD and BC = AD

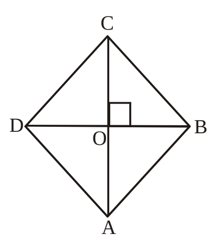
Hence, ABCD is a rectangle.

Note : In a rectangle, the diagonals are equal, i.e., AC = BD.

4 Rhombus

In a parallelogram, if all the sides are equal, then it is called a rhombus.

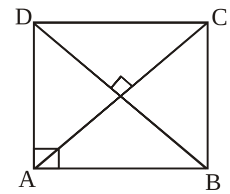
In the given figure, AB = BC = CD = DA, hence ABCD is a rhombus.



Note :

1. In a rhombus, the diagonals need not be equal.
2. In a rhombus, the diagonals bisect each other at right angles, i.e. AO = OC, BO = OD and $\overline{AC} \perp \overline{DB}$.

5 Square



In a rhombus, if each angle is a right angle, then it is called a square.

OR

In a rectangle, if all the sides are equal, then it is called a square.

In the given figure, AB = BC = CD = DA and ∠A = ∠B = ∠C = ∠D = 90º.

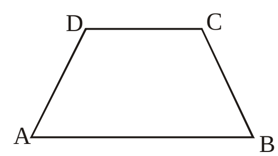
Hence, ABCD is a square.

Note :

1. In a square, the diagonals bisect each other at right angles.
2. In a square, the diagonals are equal.

6 Isosceles trapezium

In a trapezium, if the non-parallel sides are equal, then it is called an isosceles trapezium.



In the figure, $\overline{AB} \parallel \overline{CD}$ and BC = AD.

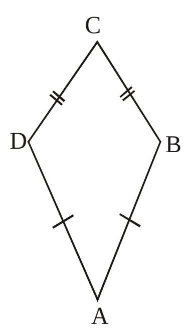
Hence, ABCD is an isosceles trapezium.

7 Kite

In a quadrilateral, if two pairs of adjacent sides are equal, then it is called a kite.

In the figure ABCD, AB = AD and BC = CD

Hence, ABCD is a kite.



NOTE :(i) Square, rectangle and rhombus are all parallelograms.

(ii) Kite and trapezium are not parallelograms.

(iii) A square is a rectangle.

(iv) A square is a rhombus.

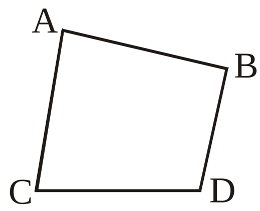
(v) A parallelogram is a trapezium.

CONVEX QUADRILATERAL:

A convex quadrilateral is four sided polygon with all its interior angles less than 180°.

In convex quadrilateral diagonals intersect inside the quadrilateral.

For example, quadrilateral ABCD is convex.

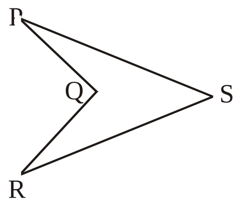


CONCAVE QUADRILATERAL :

A concave quadrilateral is four sided polygon with one interior angle more than 180°.

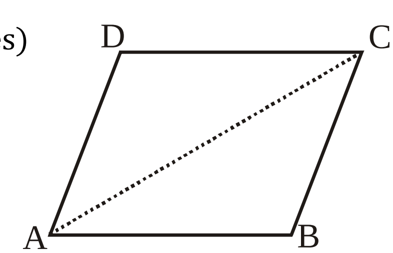
In concave quadrilateral diagonals intersect outside the quadrilateral.

For example, quadrilateral PQRS is concave.



SUM OF THE ANGLE OF A QUADRILATERAL (ANGLE SUM PROPERTY OF QUADRILATERAL) :

Consider a quadrilateral ABCD. Join A and C to get the diagonal AC which divides the quadrilateral ABCD into two triangles ABC and ADC.



We know the sum of the angles of each triangle is 180º (2 right angles)

In ΔABC, $\angle CAB + \angle B + \angle BCA = 180º$ .....(i)

In ΔADC, $\angle DAC + \angle D + \angle DCA = 180º$ .....(ii)

On adding (i) and (ii), we get

$(\angle CAB + \angle DAC) + \angle B + \angle D + (\angle BCA + \angle DCA) = 180º + 180º$

$\angle A + \angle B + \angle C + \angle D = 360º$

Thus, the sum of the angles of a quadrilateral is 360º (4–right angles)

Ex.1 The angles of a quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the quadrilateral.

Sol. Given the ratio between the angles of the quadrilateral = 3 : 5 : 9 : 13 and 3 + 5 + 9 + 13 = 30

Since, the sum of the angles of the quadrilateral = 360^0

First angle of it = $\frac{3}{30} \times 360^0 = 36^0$,

Second angle = $\frac{5}{30} \times 360^0 = 60^0$,

Third angle = $\frac{9}{30} \times 360^0 = 108^0$,

And, Fourth angle = $\frac{13}{30} \times 360^0 = 156^0$

$\therefore$ The angles of quadrilateral are 360^0, 60^0, 108^0 and 156^0.

ALTERNATE SOLUTION

Let the angles be 3x, 5x, 9x and 13.

$\therefore$ $3x + 5x + 9x + 13x = 360^0$

$\Rightarrow$ $30x = 360^0$ and $x = \frac{360^0}{30} = 12^0$

$\therefore$ 1^{st} angle = $3x = 2 \times 12^0 = 360^0$

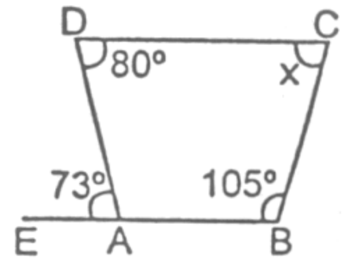
2^{nd} angle = $5x = \times 12^0 = 60^0$

3^{rd} angle = $9x = 9 \times 12^0 = 108^0$

And, 4^{th} angle = $13 \times 12^0 = 156^0$.

Ex.2 Use the informations given in adjoining figure to calculate the value of x.

Sol. Since, EAB is a straight line.



$\therefore \angle DAE + \angle DAB = 180^0$

$\Rightarrow 73^0 + \angle DAB = 180^0$

i.e., $\angle DAB = 180^0 - 73^0 = 107^0$

Since, the sum of the angles of quadrilateral ABCD is 360^0

$\therefore 107^0 + 105^0 + x + 80^0 = 360^0$

$\Rightarrow 292^0 + x = 360^0$

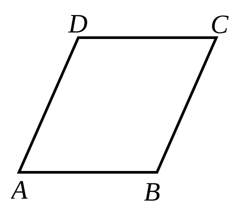
$\Rightarrow x = 360^0 - 292^0$

$\Rightarrow x = 68^0$ **Ans.**

Ex.3 In a parallelogram ABCD, prove that sum of any two consecutive angles is 180º.

Sol. Since ABCD is a parallelogram. Therefore,

AD || BC.



Now, AD || BC and transversal AB intersects them at A and B respectively.

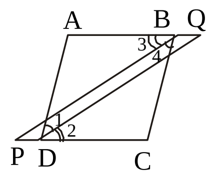
$\therefore \angle A + \angle B = 180º$

[$\because$ Sum of the interior angles on the same side of the transversal is 180º]

Similarly, we can prove that

$\angle B + \angle C = 180º$, $\angle C + \angle D = 180º$ and $\angle D + \angle A = 180º$

Ex.4 In figure bisectors of $\angle B$ and $\angle D$ of quadrilateral ABCD meet CD and AB produced at P and Q respectively. Prove that $\angle P + \angle Q = \frac{1}{2}(\angle ABC + \angle ADC)$



Sol. In ΔPBC, we have

$\therefore \angle P + \angle 4 + \angle C = 180º$

$\Rightarrow \angle P + \frac{1}{2}\angle B + \angle C = 180º$ ...(i)

In ΔQAD, we have $\angle Q + \angle A + \angle 1 = 180º$

$\Rightarrow \angle Q + \angle A + \frac{1}{2}\angle D = 180º$ ...(ii)

Adding (i) and (ii), we get

$\angle P + \angle Q + \angle A + \angle C + \frac{1}{2}\angle B + \frac{1}{2}\angle D = 180º + 180º$

$\Rightarrow \angle P + \angle Q + \angle A + \angle C + \frac{1}{2}\angle B + \frac{1}{2}\angle D = 360º$

$\Rightarrow \angle P + \angle Q + \angle A + \angle C + \frac{1}{2}(\angle B + \angle D) = \angle A + \angle B + \angle C + \angle D$

[$\therefore$ In a quadrilateral ABCD $\angle A + \angle B + \angle C + \angle D = 360º$]

$\Rightarrow \angle P + \angle Q = \frac{1}{2}(\angle B + \angle D)$

$\Rightarrow \angle P + \angle Q = \frac{1}{2}(\angle ABC + \angle ADC)$

Ex.5 In a parallelogram ABCD, $\angle D = 115º$, determine the measure of $\angle A$ and $\angle B$.

Sol. Since the sum of any two consecutive angles of a parallelogram is 180º. Therefore,

$\angle A + \angle D = 180º$ and $\angle A + \angle B = 180º$

Now, $\angle A + \angle D = 180º$

$\Rightarrow \angle A + 115º = 180º$ [$\because \angle D = 115º$ (given)]

$\Rightarrow \quad \angle A = 65^\circ \qquad$ and $\angle A + \angle B = 180^\circ$

$\Rightarrow \quad 65^\circ + \angle B = 180^\circ \Rightarrow \angle B = 115^\circ$

Thus, $\angle A = 65^\circ$ and $\angle B = 115^\circ$