TRIANGLES

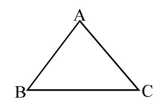
SOME PROPERTIES OF TRIANGLE

PROPERTIES OF AN ISOSCELES TRIANGLE :

An **isosceles** triangle is a triangle whose two sides are equal.

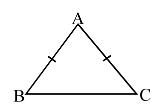
Property chart of Isosceles Triangle :

A triangle ABC is an isosceles triangle if and only if any one of the following conditions is satisfied :-



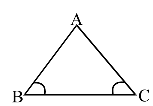
1. **Sides**

   AB = AC



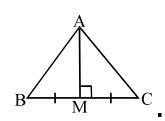
2. **Angles**

   $\angle B = \angle C$



3. **Altitude**

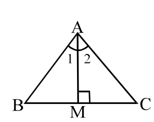
   Altitude AM bisects the side BC



4. **Altitude**

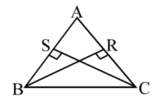
   Altitude AM bisects $\angle BAC$,

   i.e. $\angle 1 = \angle 2$



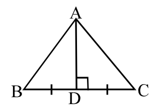
5. **Altitudes**

Altitudes BR and CS of the triangle are equal, i.e. BR = CS.



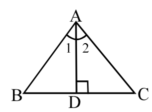
6. **Median**

Median AD is perpendicular to BC, i.e. AD ⊥ BC



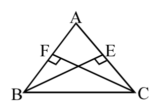
7. **Median**

Median AD bisects the ∠BAC. i.e. ∠1 = ∠2



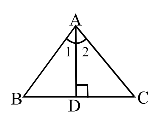
8. **Medians**

Medians BE and CF are equal, i.e. BE = CF.



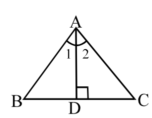
9. **Bisector**

Bisector AD of ∠BAC bisects the side BC, i.e. BD= CD



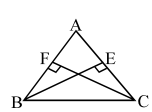
10. **Bisector**

Bisector AD of ∠BAC is prependicular to the side BC i.e. AD ⊥ BC

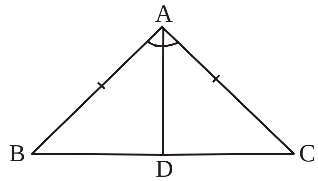


11. **Bisectors**

Bisectors of ∠B and ∠C are equal in length i.e. BE= CF



Theorem 1: The angles opposite to equal sides of a triangle are equal.



Proof:

We are given an isosceles triangle ABC in which AB = AC.

We need to prove that $\angle B = \angle C$.

Let us draw the bisector of $\angle A$ and let D be the point of intersection of this bisector of $\angle A$ and BC (see in figure)

In ΔBAD and ΔCAD

AB = AC [Given]

$\angle BAD = \angle CAD$ [By construction]

AD = AD [Common]

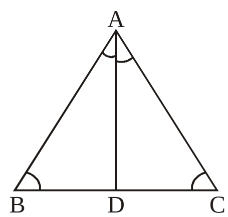
So, $\Delta BAD \cong \Delta CAD$ [By SAS rule]

So, $\angle ABD = \angle ACD$, since they are corresponding angles of congruent triangles.

So, $\angle B = \angle C$.

Theorem 2: If two angles of a triangle are equal, then the sides opposite to them are also equal.

Theorem : The sides opposite to equal angles of a triangle are equal.



Given : ΔABC, in which $\angle B = \angle C$

To Prove : AB = AC

Construction : Draw AD, the bisector of angle $\angle BAC$ which meets BC at D.

Proof : In ΔABD and ΔACD,

$\angle B = \angle C$ (Given)

AD = AD (Common side)

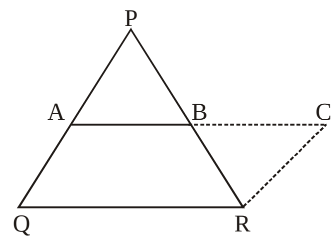
∠BAD = ∠CAD (By construction)

Therefore, ΔABD ≅ ΔACD (By AAS)

Hence corresponding sides, AB = AC

Theorem 3 (Mid-Point Theorem) :

In a triangle, the line segment joining the mid-points of any two sides is parallel to the third side and also half of it.



Given : In ΔPQR, A and B are the mid-points of $\overline{PQ}$ and $\overline{PR}$ respectively.

To Prove : $\overline{AB} \parallel \overline{QR}$ and AB = $\frac{1}{2}$ QR

Construction : Draw $\overline{RC}$ parallel to $\overline{QA}$ to meet produced at $\overline{AB}$ C.

Proof :

(i) In ΔABP and ΔCBR,

∠PBA = ∠RBC (vertically opposite angles)

∠PAB = ∠RCB (alternate angles and $\overline{CR} \parallel \overline{PQ}$)

PB = BR (B is mid point of PR)

By AAS congruence property,

ΔABP ≅ ΔCBR

PA = CR and AB = BC (corresponding parts of congruent triangles)

⇒ AQ = CR (PQ = 2AQ)

In quadrilateral ACRQ, AQ = CR and $\overline{AQ} \parallel \overline{CR}$

∴ ACRQ is a parallelogram.

∴ $\overline{AC} \parallel \overline{QR}$

⇒ $\overline{AC} \parallel \overline{QR}$

(ii) AC = QR (opposite sides of parallelogram)

⇒ QR = AB + BC

$\Rightarrow$ QR = 2AB (AB = BC)

$\Rightarrow$ AB = $\frac{1}{2}$QR

Ex.1 Find ∠BAC of an isosceles triangle in which AB = AC and ∠B = $\frac{1}{3}$ of right angle.

Sol. $\angle B = \angle C = \frac{1}{3}(90) = 30°$

$\therefore \angle A + \angle B + \angle C = 180°$ (λ.p.)

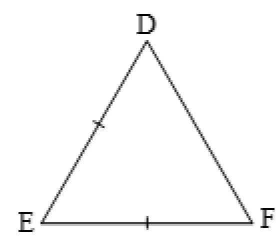
$\angle A + 30° + 30° = 180° \Rightarrow \angle A = 120°$.

Ex.2 In isosceles triangle DEF, DE = EF and ∠E = 70° then find other two angles.

Sol. Let ∠D = ∠F = x

$\therefore \angle D + \angle E + \angle F = 180°$

(angle sum property)



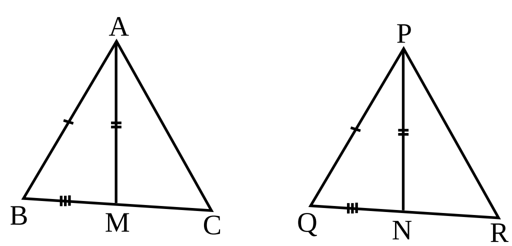
$\Rightarrow$ x + 70° + x = 180°

$\Rightarrow$ 2x = 110°

$\Rightarrow$ x = 55°

$\Rightarrow \angle D = \angle F = 55°$.

Ex.3 Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of ΔPQR (see figure). Show that:



(i) ΔABM ≅ ΔPQN (ii) ΔABC ≅ ΔPQR

Sol.(i) In ΔABM & ΔPQN

AB = PQ (given)

AM = PN (given)

BM = QN $\left(\because\ BC = Q\ \therefore\ \frac{BC}{2} = \frac{QR}{2}\right)$

$\therefore$ By SSS, $\Delta ABM \cong \Delta PQN$ Proved.

(ii) In ΔABC & ΔPQR

AB = PQ (given)

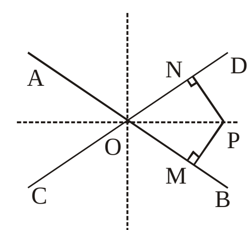
$\angle B = \angle Q$ (c.p.c.t. of part (i))

BC = QR (given)

$\therefore$ By SAS, $\Delta ABC \cong \Delta PQR$ Proved.

Ex.4 Prove that any point on the bisector of an angle is equidistant from the arms of the angle.

Sol. **Given :** Two straight lines AB and CD intersect at O, P is a point on the bisector of $\angle BOD$,



Construction: Draw $MP \perp OB$ and $NP \perp OD$

In ΔOMP and ΔONP,

$\angle MOP = \angle NOP$ [P lies on the bisector $\angle BOD$]

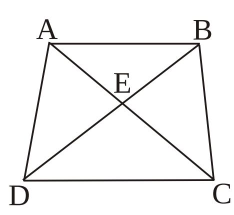
$\angle OMP = \angle ONP$ [each is 90º]

OP = OP [Common]

$\Delta OMP \cong \Delta ONP$ [by AAS congruence rule]

MP = NP [by CPCT]

Ex.5 In fig. AD = BC and BD= CA, Prove that ADB = BCA and DAB = CBA



Sol In triangles ABD & ABC,

We have :

AD = BC [given]

BD = CA [given]

AB = AB [common]

So, by SSS congruence criterion, we have :

$\Delta ABD \cong \Delta BAC$

$\angle DAB = \angle CBA$ [Since, corresponding parts of congruent triangles are equal]

$\angle ADB = \angle BCA$