TRIANGLES

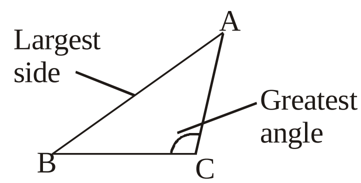
INEQUALITIES IN TRIANGLE

SIDES AND ANGLES OF A TRIANGLE :

Theorem 4 :

If two sides of a triangle are unequal, the larger side has the greater angle opposite to it.

The converse of the above property is also true.



Deduction :

1. In any triangle, the greater angle has the larger side opposite to it.
2. In any triangle, the largest side is opposite to the greatest angle.
3. Angles opposite to equal sides are equal and vice versa. If $\angle A = \angle B$, then $BC = CA$
   If $BC = CA$, then $\angle A = \angle B$.
4. If the angles are in increasing or decreasing order, sides opposite to them also will be in the same order.

(i) If $\angle A > \angle B > \angle C$, then $BC > CA > AB$.

(ii) If $\angle A < \angle B < \angle C$, then $BC < CA < AB$.

Ex.1 In ΔABC, $AB = 10$cm and $BC = 8$ cm. Find the range of values that CA can take.

Sol: In a triangle, the sum of two sides is greater than the third side and the difference of two sides is less than the the third side.

$CA < AB + BC$ and $CA > AB - BC$

$\Rightarrow$ $CA < 18$ cm and $CA > 2$ cm

$\Rightarrow$ $2 \text{ cm} < CA > 18$ cm.

SUM AND DIFFERENCE OF TWO SIDES OF TRIANGLE :

Theorem 5

The sum of two sides of a triangle is always greater than the third side.

Thus, $AB + AC > CB$

AC + BC > AB

BC + AB > AC

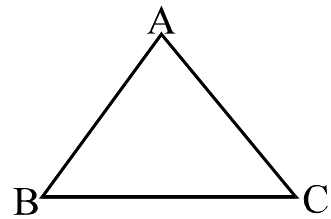
Deduction:

The difference of two sides is always less than the third side.

AB > AC – BC and BC – AC

AC > AB – BC and BC – AB

BC > AC – AB and AB – AC



1 Perpendicular Line Segment is the Shortest :

Of all the line segments that can be drawn to a given line *l* from a point O outside the line *l*, the perpendicular line segment is the shortest. That is :

OP < OA, OB, OC, OD..... etc.

Proof:

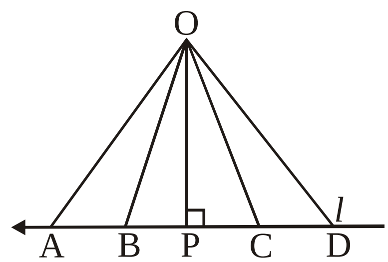
Suppose OP ⊥ *l*.

Let OC be any line other than OP.

In ΔOPC,

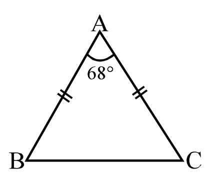
∠OPC = 90º > ∠ OCP

Therefore, OP < OC (The side opposite to greater angle is larger)



Ex.2 In the figure given below, AB= AC . Is AB > BC ?

Sol:



As, AB = AC, ∠B = ∠C [Angles opposite to equal sides are equal]

Also, ∠A + ∠B + ∠C = 180º [Sum of the angles of a triangle]

⇒ 68° + ∠C + ∠C = 180° [Since ∠B = ∠C)

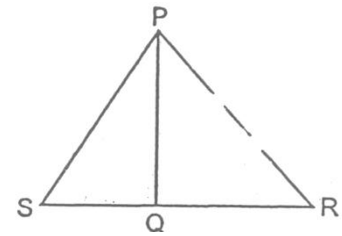
⇒ 2∠C = 180 – 68

⇒ Now, as ∠A = 68° > ∠C = 56°

⇒ BC > AB [Since, greater angle has the larger side opposite to it]

Thus, AB < BC.

Ex.3 In figure, PQ = PR, show that PS > PQ



Sol. In ΔPQR

$\therefore$ PQ = PR

$\Rightarrow \angle PRQ = \angle PQR$ ....(i) [Angles opposite to equal sides]

In ΔPSQ, SQ is produced to R

$\therefore$ Ext. $\angle PQR > \angle PSQ$ ....(ii)

$\angle PRQ > \angle PSQ$ [By eq. (i) and (ii)]

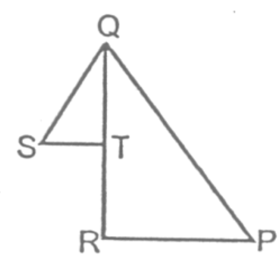
$\Rightarrow \angle PRS > \angle PSR$

$\Rightarrow$ PS > PR [Sides opposite to greater angles is larger]

But, PR = PQ

$\therefore$ PS > PQ **Hence Proved.**

Ex.4 In figure, T is a point on side QR of ΔPQR and S is a point such that RT = ST. Prove that PQ + PR > QS



Sol. In ΔPQR we have

PQ + PR > QR

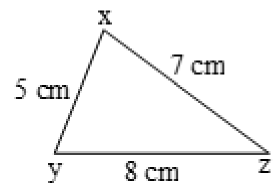
$\Rightarrow$ PQ + PR > QT + TR

$\Rightarrow$ PQ + PR > QT + ST $\therefore$RT = ST

In ΔQST QT + ST > SQ

$\therefore$ PQ + PR > SQ

Ex.5 Find the relation between angles in figure.



Sol. $\because$ yz > xz > xy

$\therefore \angle x > \angle y > \angle z$.

($\because$ Angle opposite to longer side is greater)