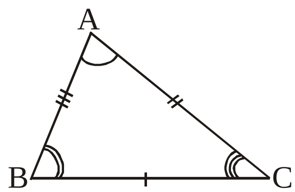
LINES AND ANGLES

TRIANGLE AND THEIR THEOREM

TRIANGLE : Triangle is a three sided closed polygon.

Six Elements of a Triangle : Three sides BC, CA and AB. Three Angles A, B and C.

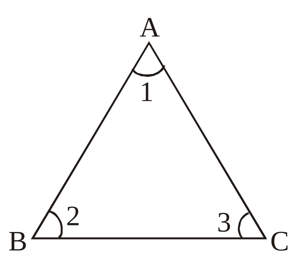


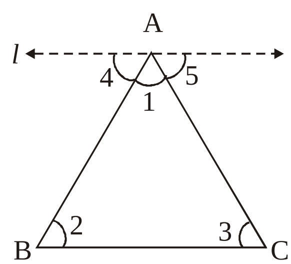
Relation Between the Angles of a Triangle :

Theorem 7 :

The sum of the three angles of a triangles is 180º. This is called the angle sum property of triangle.

Given : A triangle ABC





To Prove :

$\angle A + \angle B + \angle C = 180^\circ$ i.e. $\angle 1 + \angle 2 + \angle 3 = 180^\circ$

Construction : Through A, draw a line l parallel to BC.

Proof :

Since $l \parallel BC$

$\angle 2 = \angle 4$ .....(i) [Alternate interior angles]

and $\angle 3 = \angle 5$ .....(ii) [Alternate interior angles]

Adding (i) and (ii) we get :

$\Rightarrow \quad \angle 2 + \angle 3 = \angle 4 + \angle 5$?

$\Rightarrow \quad \angle 1 + \angle 2 + \angle 3 = \angle 1 + \angle 4 + \angle 5$ [Adding 1 to both the sides]

$\Rightarrow \quad \angle 1 + \angle 2 + \angle 3 = \angle 4 + \angle 1 + \angle 5$

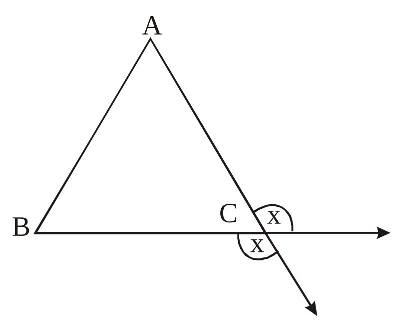
$\Rightarrow \quad \angle 1 + \angle 2 + \angle 3 = 180^\circ$ [sum of angles at a point on a line is 180º]

That is $\angle A + \angle B + \angle C = 180^\circ$

Thus, the sum of the three angles of a triangle is 180º.

Exterior Angle of a Triangle :

In fig. $\angle ACB$ is called the interior angle C. Either of the angles marked x (they are equal. vertically opposite) is called the corresponding exterior angle.



Angles A and B are called the remote interior angles or interior opposite angles of the angle x.

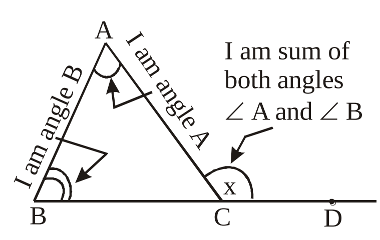
Theorem 8 :

(Exterior Angle Theorem) : If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior oppsoite angles.

Corollary :

$\angle ACD > \angle A$ and $\angle ACD > \angle B$

Thus, an exterior angle of a triangle is greater than either of the interior oppsite angles.



Ex.1 One of the angles of a triangle is 65º. Find the remaining two angles, if their difference is 25º.

Sol. Let $\angle A = 65^\circ$ and two other angles be x and y.

Then we are given that $x - y = 25^\circ$ .....(1)

we know that : $\angle A + \angle B + \angle C = 180^\circ$

or $65^\circ + x + y = 180^\circ$

$x + y = 115^\circ$ .....(2)

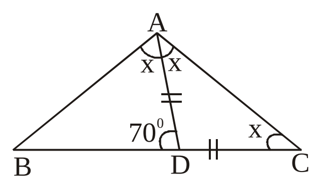
Adding (1) and (2), we get :

$2x = 140^\circ \Rightarrow x = 70^\circ$

From (2), we get, $y = 115^\circ - x = 115^\circ - 70^\circ = 45^\circ$

Thus the remaining two angles of the triangle are 70º and 45º.

Ex.2 In ΔABC, AD is the bisector of $\angle BAC$ and AD = DC. If $\angle BDA = 70^\circ$, find $\angle A$, $\angle B$ and $\angle C$.



Sol In Δ DAC

DA = DC $\Rightarrow$ Δ DAC is an isosceles triangle.

$\angle DAC = \angle DCA = x$ (say)

Also, $\angle ADB = \angle DAC + \angle DCA = x + x = 2x$

[Exterior angle is equal to sum of the two opposite interior angles]

$\Rightarrow$ $70^\circ = 2x$

$\Rightarrow$ $x = 35^\circ$

Next, as AD is the bisector of $\angle BAC$,

$\angle BAD = \angle DAC = x = 35^\circ$

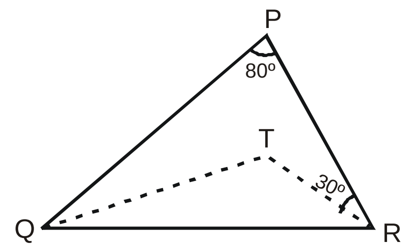
Thus, $\angle A = \angle BAD + \angle DAC$ $\Rightarrow$ $35^\circ + 35^\circ = 70^\circ$

$\therefore$ $\angle A = 70^\circ$, $\angle C = 35^\circ$

and $\angle B = 180^\circ - \angle A - \angle C$ $\Rightarrow 180^\circ - 70^\circ - 35^\circ = 75^\circ$

Ex.3 In figure, TQ and TR are the bisectors of $\angle Q$ and $\angle R$ respectively. If $\angle QPR = 80^\circ$ and $\angle PRT = 30^\circ$, determine $\angle TQR$ and $\angle QTR$.

Sol. Since the bisectors of $\angle Q$ and $\angle R$ meet at T.



$\therefore \quad \angle QTR = 90^\circ + \angle QPR$

$\Rightarrow \quad \angle QTR = 90^\circ + (80^\circ)$

$\Rightarrow \quad \angle QTR = 90^\circ + 40^\circ = 130^\circ$

In ΔQTR, we have

$\angle TQR + \angle \Theta TR + \angle TRQ = 180^\circ$

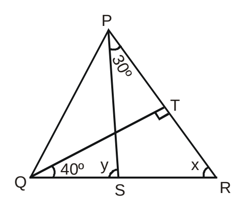
$\Rightarrow \quad TQR + 130^\circ + 30^\circ = 180^\circ$ [Θ $\angle TRQ = \angle PRT = 30^\circ$]

$\Rightarrow \quad \angle TQR = 20^\circ$

Thus, $\angle TQR = 20^\circ$ and $\angle QTR = 130^\circ$.

Ex. 4 In figure , If $QT \perp PR$, $\angle TQR = 40^\circ$ and $\angle SPR = 30^\circ$, find x and y.

Sol. In ΔTQR



$\angle TQR + \angle QTR + \angle TRQ = 180^\circ$

$\Rightarrow \quad 40^\circ + 90^\circ + \angle TRQ = 180^\circ$

$\Rightarrow \quad \angle TRQ = 180^\circ - 130^\circ = 50^\circ$

$\Rightarrow \quad x = 50^\circ$

In ΔPSR, using exterior angle property, we have

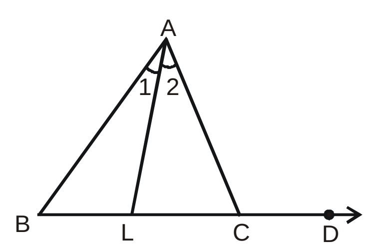
$\angle PSQ = \angle PRS + \angle RPS$

$\Rightarrow \quad y = x + 30^\circ$

$\Rightarrow \quad y = 50^\circ + 30^\circ = 80^\circ$.

Ex. 5 The side BC of a ΔABC is produced, such that D is on ray BC. The bisector of $\angle A$ meets BC in L as shown in figure. Prove that $\angle ABC + \angle ACD = 2\angle ALC$.

Sol. In ΔABC, we have



ext. $\angle ACD = \angle B + \angle A$

$\Rightarrow \quad$ ext. $\angle ACD = \angle B + 2\angle 1 \quad$ ...(i)

[$\because$ AL is the bisector of $\angle A$ $\because$ $\angle A = 2\angle 1$]

$\Rightarrow \quad \angle ACD = \angle B + 2\angle 1$

In ΔABL, we have

ext. $\angle ALC = \angle B + \angle BAL$

$\Rightarrow$ ext. $\angle ALC = \angle B + \angle 1$

$\Rightarrow$ $2\angle ALC = 2\angle B + 2\angle 1$ ...(ii) [Multiplying both sides by 2]

Subtracting (i) from (ii), we get

$2\angle ALC - \angle ACD = \angle B$

$\Rightarrow$ $\angle ACD + \angle B = 2\angle ALC$

$\Rightarrow$ $\angle ACD + \angle ABC = 2\angle ALC$.