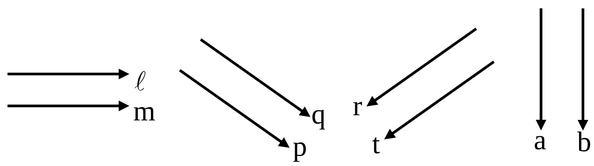
# LINES AND ANGLES

## PARALLEL LINE, TRANSVERSAL LINE AND THEIR THEOREM

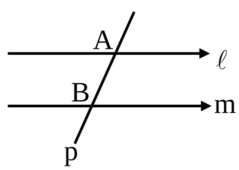
### Parallel lines:

The lines which are in same plane and do not intersect each other an y where, i.e. distance between parallel lines is same anywhere




### Transversal line :

A line which intersect two or more given lines at distinct points, is called a transversal of the given lines.




Here λ || m and p is transversal line.

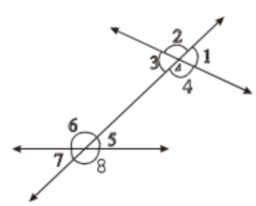
Part of transversal which is between the two lines is called intercept (AB).

### Angles made by a transversal on two parallel lines :

### Transversal :

A transversal is a line that intersects two or more lines at different points.

A transversal cutting two lines forms some angles that are given special names.



(i) Interior Angles : $\angle 3, \angle 4, \angle 5$ and $\angle 6$

(ii) Exterior Angles : $\angle 1, \angle 2, \angle 7$ and $\angle 8$

(iii) Pairs of Corresponding Angles : $\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$, $\angle 4$ and $\angle 8$

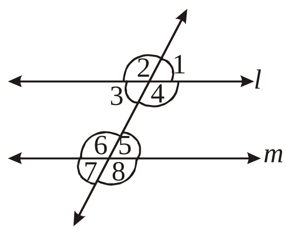
(iv) Pairs of Alternate Interior Angles : $\angle 3$ and $\angle 5$, $\angle 4$ and $\angle 6$

(v) Pairs of Alternate Exterior Angles : $\angle 1$ and $\angle 7$, $\angle 2$ and $\angle 8$

(vi) Consecutive Interior Angles : $\angle 3$ and $\angle 6$, $\angle 4$ and $\angle 5$.

(vii) Pair of co-interior angles : $\angle 4$ & $\angle 5$, $\angle 3$ & $\angle 6$.

(viii) Pair of co-exterior angles : $\angle 1$ & $\angle 8$, $\angle 2$ & $\angle 7$.




**Axiom :** (Corresponding Angles Axiom)

If a transversal intersects two parallel lines, then each pair of corresponding angles are equal. That is, in Fig.

$\angle 1 = \angle 5$, $\angle 2 = \angle 6$, $\angle 3 = \angle 7$ and $\angle 4 = \angle 8$

**Axiom :**

If a transversal intersects two lines such that a pair of corresponding angles is equal, then the two lines are parallel to each other.

In Fig. if $\angle 1 = \angle 5$ or $\angle 2 = \angle 6$ or $\angle 3 = \angle 7$ and $\angle 4 = \angle 8$, then lines $l$ and $m$ are parallel.

**Theorem 2 :**

If a transversal intersects two parallel lines, then each pair of alternate interior angles is equal. i.e., if $l \,||\, m$, $\angle 3 = \angle 5$ and $\angle 4 = \angle 6$.

**Theorem 3 :**

If a transversal intersects two lines in such a way that a pair of alternate exterior angles is equal, then the lines are parallel. i.e., if $\angle 1 = \angle 7$ and $\angle 2 = \angle 8 \Rightarrow l \,||\, m$.

## Theorem 4 :

If a transversal intersects two parallel lines, then each pair of interior angles on the same side of the transversal is supplementary. i.e., if $\angle 4 + \angle 5 = 180º$ and $\angle 3 + \angle 6 = 180º \Rightarrow l \,||\, m$.

## Theorem 5 :

If a transversal intersects two lines in such a way that a pair of exterior angles
on the same side of the transversal is supplementary, then the lines are parallel. i.e.,
if $\angle 1 + \angle 8 = 180º$ and $\angle 2 + \angle 7 = 180º \Rightarrow l \,||\, m$.

## Important Facts to Remember :

(i) If a ray stands on line, then the sum of the adjacent angles so formed is $180^0$.

(ii) If the sum of two adjacent angles is $180^0$, then their non common arms are two apposite rays.

(iii) The sum of all the angles round a point is equal to $360^0$

(iv) If two lines intersect, then the vertically opposite angles are equal.

(v) If a transversal interests two parallel lines then the corresponding angles are equal, each pair of alternate interior angles are equal and each pair of consecutive interior angles are supplementary.

(vi) if a transversal intersects two lines in such a way that a pair of alternet interior angles are equal, then the two lines are parallel.

(vii) If a transversal intersects two lines in such a way that a pair of consecutive interior angles are supplementary, then the two lines are parallel.

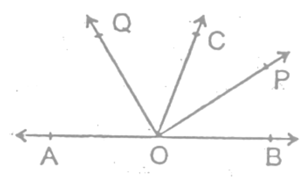
(viii) If two parallel lines are intersected by a transversal, the bisectors of any pair of alternate interior angles are parallel and the bisectors of an two corresponding angles are also parallel.

(ix) If a line i s perpendicular to one or two given parallel, lines, then it is also perpendicular to the other line.

(x) Two angles which have their arms parallel are either equal or supplementary.

(xi) Two angles whose arms are perpendicular are either equal or supplementary.

**Ex.1** In figure, OP and OQ bisects $\angle BOC$ and $\angle AOC$ respectively. Prove that $\angle POQ = 90^0$.




**Sol.** $\therefore$ OP bisects $\angle BOC$

$\therefore \angle POC = \frac{1}{2}\angle BOC$ ...(i)

Also OQ bisects $\angle AOC$

$\therefore \angle COQ = \frac{1}{2}\angle AOC$ ...(ii)

$\therefore$ OC stands on AB

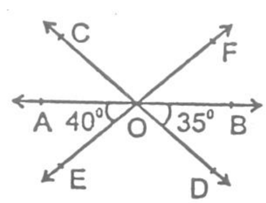
$\therefore \angle AOC + \angle BOC = 180^0$ [Linear pair]

$\Rightarrow \frac{1}{2}\angle AOC + \frac{1}{2}\angle BOC = \frac{1}{2} \times 180^0$

$\Rightarrow \angle COQ + \angle POC = 90^0$ [Using (i) & (ii)]

$\Rightarrow \angle POQ = 90^0$ [By angle sum property]

**Ex.2** In figure, lines AB, CD and EF intersect at O. Find the measures of $\angle AOC$, $\angle DOE$ and $\angle BOF$



**Sol.** Given $\angle AOE = 40^0$ & $\angle BOD = 35^0$

Clearly $\angle AOC = \angle BOD$ [Vertically opposite angles]

$\Rightarrow \angle AOC = 35^0$ **Ans.**

$\angle BOF = \angle AOE$ [Vertically opposite angles]

$\Rightarrow \angle BOF = 40^0$ **Ans.**

Now, $\angle AOB = 180^0$ [Straight angles]

$\Rightarrow \angle AOC + \angle COF + \angle BOF = 180^0$ [Angles sum property]

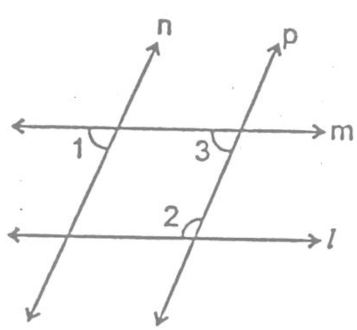
$\Rightarrow 35^0 + \angle COF + 40^0 = 180^0$

$\Rightarrow \angle COF = 180^0 - 75^0 = 105^0$

Now, $\angle DOE = \angle COF$ [Vertically opposite angles]

$\therefore \angle DOE = 105^0$ **Ans.**

**Ex.3** In figure if I $\parallel$ m , n $\parallel$ p and $\angle 1 = 85^0$ find $\angle 2$




**Sol.** $\therefore$ n $\parallel$ p and m is transversal

$\therefore \angle 1 = \angle 3 = 85^0$ [Corresponding angles]

Also m $\parallel$ I & p is transversal

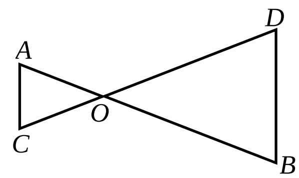
$\therefore \angle 2 + \angle 3 = 180^0$ [ $\because$ Consecutive interior angles]

$\Rightarrow \angle 2 + 85^0 = 180^0$

$\Rightarrow \angle 2 + 180^0 - 85^0$

$\Rightarrow \angle 2 = 95^0$ **Ans.**

**Ex.4** In figure given that ∠AOC = ∠ACO and ∠BOD = ∠BDO. Prove that AC || DB.




**Sol.** We have,

∠AOC = ∠ACO and ∠BOD = ∠BDO

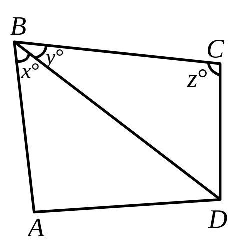
But ∠AOC = ∠BOD [Vertically opp. ∠s]

∴ ∠ACO = ∠BOD and ∠BOD = ∠BDO

⇒ ∠ACO = ∠BDO

Thus, AC and BD are two lines intersected by transversal CD such that ∠ACO = ∠BDO i.e. alternate angles are equal. Therefore, AC || DB.

**Ex.5** In figure AB || DC if $x = \frac{4}{3}y$ and $y = \frac{3}{8}z$, find the values of x, y and z.




**Sol.** Since AB || DC and transversal BD intersects them at B and D respectively.

Therefore,

∠ABD = ∠CDB ⇒ ∠CDB = x°

In ΔBCD, we have

y° + z° + x° = 180°

$\Rightarrow \frac{3}{8}z^{º} + z^{º} + \frac{4}{3} \times \frac{3}{8}z^{º} = 180$

$\Rightarrow \frac{3}{8}z^{º} + z^{º} + \frac{1}{2}z^{º} = 180$

$\Rightarrow \frac{15}{8}z^{º} = 180^{º}$

$[\because x = \frac{4}{3}y$ and $y = \frac{3}{8}z$

$\therefore x = \frac{4}{3} \times \frac{3}{8}z = \frac{z}{2}]$

$\Rightarrow z^{º} = 180^{º} \times \frac{8}{15} = 96^{º}$

Now, $y = \frac{3}{8}z \Rightarrow y = \frac{3}{8} \times 96^{º} = 36^{º}$

and $x = \frac{4}{3}y \Rightarrow x = \frac{4}{3} \times 36^{º} = 48$