LINES AND ANGLES

BASIC TERMS AND DEFINITION OF LINES AND ANGLES

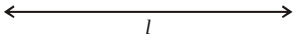
INTRODUCTION :

You have already studied about points, lines and angles in our earlier classes. In this chapter, you will study the properties of the angle formed when two lines intersect each other, and also the properties of the angles formed when a line intersects two or more parallel lines at distinct points.

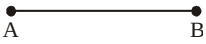
LINE, LINE-SEGMENT AND RAY :

Line is a set of infinite points that has no end point. If A and B are any two points on line *l* then we denote the line as $\overleftrightarrow{AB}$ and is represented as

We also denote the line by a single small English Alphabet say *l* and represented as



Line-Segment is a part of a line that has two end points. It is of finite length and cannot be extended further. Line segment is denoted as $\overline{AB}$ where A, B are its end points and is represented as



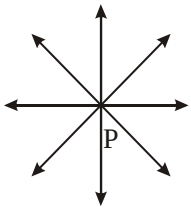
Ray is a part of a line with one end point. It can be extended infinitely on one side. Ray is denoted as where A is the end point and B is any other point on the ray is represented as

Here B is not the end point

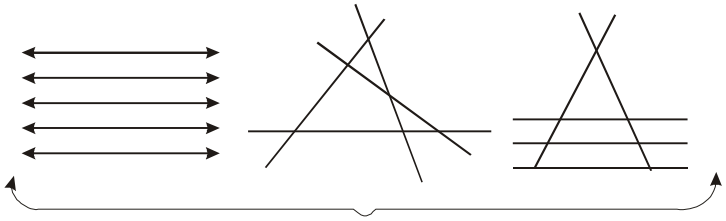
We can extend the ray towards B from A infinitely.

Concurrent lines

Three or more distinct lines are said to be concurrent if each of them passes through the same point.



Concurrent line

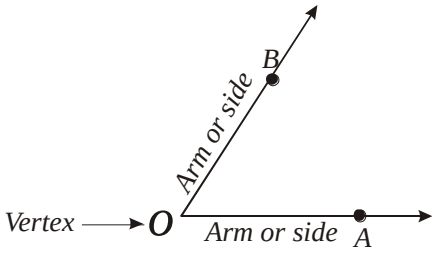


current lines Non-concurrent lines

ANGLES AND THEIR PROPERTIES :

Angle : An angle is figure formed by two rays with a common intitial point. The common initial point is called the vertex of the angle, and the rays forming the angle are called arms or sides of the angle.

In fig. O, is the vertex and OA and OB are sides of the angle AOB or BOA.



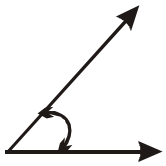
We also use the symbol ' $\angle$ ' to denote the measure of the angle.

Angular Region :

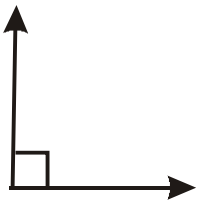
It consists of the angle AOB and its interior.

Different types of Angles :

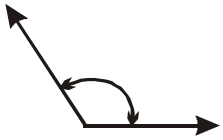
(i) **Acute angle :** An acute angle is an angle whose measure lies between 0º and 90º.



(ii) **Right angle :** A right angle which is exactly 90º. The symbol " " is used to indicate a right angle.

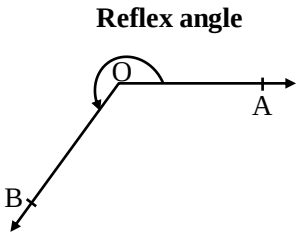


(iii) **Obtuse angle :** A obtuse angle is an angle measuring greater than 90º but less than 180º.



(iv) **Straight Angle :** A straight angle is an angle which is exactly 180º.

(v) **Reflex angle :** A reflex angle is greater than 180º but less than 360º.



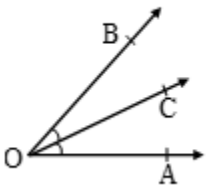
(v)

Bisector of an angle:

A ray OC is called the bisector of ∠AOB,

if m∠AOC = m∠BOC.

In this case, $\angle AOC = \angle BOC = \frac{1}{2}\angle AOB$.



Complementary angles:

Two angles are said to be complementary, if the sum of their measures is 90°.

Two complementary angles are called the complement of each other.

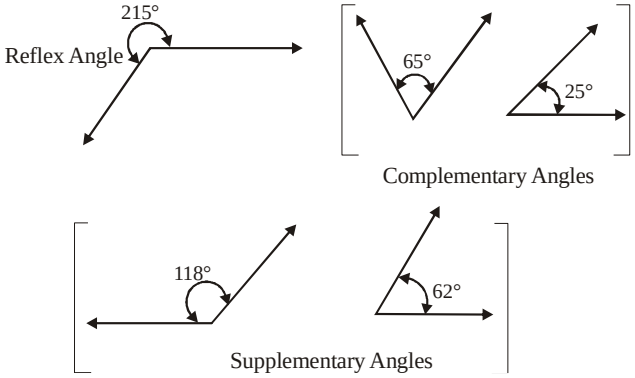
Ex. : Angles measuring 20° and 70° are complementary angles.

Supplementary angles:

Two angles are said to be supplementary, if the sum of their measures is 180°.

Two supplementary angles are called the supplement of each other.

Ex. : Angles measuring 60° and 120° are supplementary angles.



Ex.1 Two supplementary angles are in ratio 4 : 5, find the angles,

Sol. Let angles are 4x & 5x.

$\therefore$ Angles are supplementary

$\therefore \quad 4x + 5x = 180^0 \Rightarrow 9x = 180^0$

$\Rightarrow \quad x = \frac{180^0}{9} = 20^0$

$\therefore$ Angles are 4×20^0, 5×20^0 80^0 & 100^0 **Ans.**

Ex.2 If an angle differs from its complement by 10, find the angle.

Sol. let angles is x^0 then its complement is $90 - x^0$.

Now given $x^0 - (90 - x^0) = 10$

$\Rightarrow x^0 - 90^0 + x^0 = 10$

$\Rightarrow 2x^0 = 10 + 90 = 100$

$\Rightarrow x^0 = \frac{100^0}{2} = 50^0$

$\therefore$ Required angle is 50^0.

Ex.3 An angle is equal to one-third of its supplement. Find its measure.

Sol. Let the measure of the required angle be x degrees. Then,

Its supplement = $180^\circ - x$. It is given that :

Angle = $\frac{1}{3}$ (Its supplement)

$\Rightarrow x = \frac{1}{3}(180^\circ - x) \Rightarrow 3x = 180^\circ - x$

$\Rightarrow 4x = 180^\circ \quad \Rightarrow \quad x = 45^\circ$

Thus, the measure of the given angle is 45°.

Ex.4 Two supplementary angles are in the ratio 2 : 3. Find the angles.

Sol. Let the two angles be 2x and 3x in degrees. Then,

$2x + 3x = 180^\circ$

$\Rightarrow 5x = 180^\circ \quad \Rightarrow x = 36^\circ$

Thus, the two angles are $2x = 2 \times 36^\circ = 72^\circ$

and $3x = 3 \times 36^\circ = 108^\circ$

Ex.5 Write the complement of the following angles: $30^\circ 20'$

Sol. Complement of

$30^\circ 20' = 90^\circ - 30^\circ 20'$

$= 90^\circ - (30^\circ + 20')$

$= (89^\circ - 30^\circ) + (1^\circ - 20')$

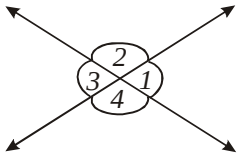
$= 59º + (60′ - 20′)$ $[\because 1º = 60′]$

$= 59º + 40′ = 59º\ 40′$

Vertically Opposite Angles :

In figure two intersecting lines form the angles numbered 1,2,3 and 4.

Angles 1 and 3 are called vertically opposite angles as are angles 2 and 4.



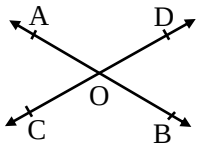
Theorem : If two lines intersect then the vertically opposite angles are equal.

Given : Two lines AB and CD intersect at a point O.

To prove : (i) $\angle AOC = \angle BOD$, (ii) $\angle AOD = \angle BOC$

Proof :

Since ray OA stands on line CD, we have:



$\angle AOC + \angle AOD = 180°$ [linear pair].

Again, ray OD stands on line AB.

$\therefore\ \angle AOD + \angle BOD = 180°$ [linear pair]

$\therefore\ \angle AOC + \angle AOD = \angle AOD + \angle BOD$

[each equal to 180°]

$\therefore\ \angle AOC = \angle BOD$

Similarly, $\angle AOD = \angle BOC$