LINES AND ANGLES

ADJACENT ANGLE, LINEAR PAIR, VERTICALLY OPPOSITE ANGLES

Adjacent Angles :

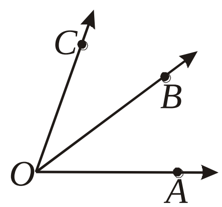
Two angles AOB and BOC are such that :

(i) they have a common vertex.

(ii) have a common arm and

(iii) their interiors are non-overlapping.

Two angles satisfying the above conditions are called adjacent angles.

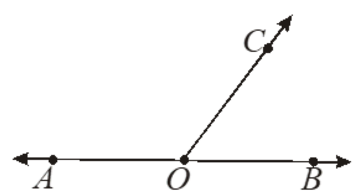


Linear Pair :

Two adjacent angles whose non common arms are opposite ray are said to form a linear pair.

$\angle AOC$ and $\angle BOC$ form a linear pair.

$\angle AOC + \angle BOC = 180^\circ$

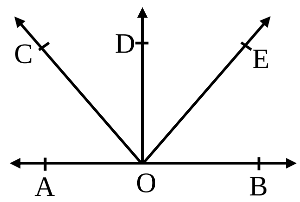


Theorem 1 :

Prove that the sum of all the angles formed on the same side of a line at a given point on the line is 180°.

Given :

AOB is a straight line and rays OC, OD and OE stand on it, forming $\angle AOC$, $\angle COD$, $\angle DOE$ and $\angle EOB$.



To prove :

$\angle AOC + \angle COD + \angle DOE + \angle EOB = 180°$.

Proof :

Ray OC stands on line AB.

$\therefore \quad \angle AOC + \angle COB = 180°$

$\Rightarrow \angle AOC + (\angle COD + \angle DOE + \angle EOB) = 180°$

$[\angle COB = \angle COD + \angle DOE + \angle EOB]$

$\Rightarrow \angle AOC + \angle COD + \angle DOE + \angle EOB = 180°$.

Hence, the sum of all the angles formed on the same side of line AB at a point O on it is 180°.

Theorem 2 :

Prove that the sum of all the angles around a point is 360°.

Given : A point O and the rays OA, OB, OC, OD and OE make angles around O.

To prove :

$\angle AOB + \angle BOC + \angle COD + \angle DOE + \angle EOA = 360°$

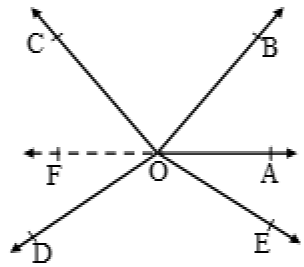
Construction : Draw a ray OF opposite to ray OA.

Proof :

Since ray OB stands on line FA, we have : $\angle AOB + \angle BOF = 180°$ [linear pair]

$\therefore \quad \angle AOB + \angle BOC + \angle COF = 180° \quad ....(i)$

[$\because \angle BOF = \angle BOC + \angle COF$]



Again, ray OD stands on line FA.

$\therefore\ \angle FOD + \angle DOA = 180°$ [linear pair]

or $\angle FOD + \angle DOE + \angle EOA = 180°$ ...(ii)

[$\because \angle DOA = \angle DOE + \angle EOA$]

Adding (i) and (ii), we get :

$\angle AOB + \angle BOC + \angle COF + \angle FOD + \angle DOE + \angle EOA = 360°$

$\therefore\ \angle AOB + \angle BOC + \angle COD + \angle DOE + \angle EOA = 360°$

[$\because\ \angle COF + \angle FOD = \angle COD$]

Hence, the sum of all the angles around a point O is 360°.

IMPORTANT POINTS

- Two angles are called adjacent angles if

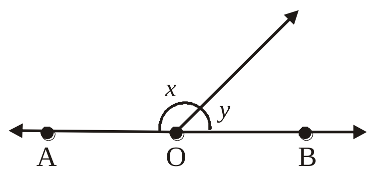
(i) they have the same vertex,

(ii) they have a common arm , and

(iii) uncommon arms are on either side of the common arm.

- Two adjacent angles are said to form a linear pair of angles, if the non-common arms are two opposite rays.
- If a ray stands on a line, then the sum of the adjacent angles so formed is 180º.
- If the sum of two adjacent angles is 180º, then their non-common arms are two opposite rays.
- The sum of all the angles round a point is equal to 360º.
- Two angle are called a pair of vertically opposite angles, if their arms form two pairs of opposite rays.
- If two lines intersect, then the vertically opposite angles are equal.

Ex.1 In figure, AB is a straight line and x is greater than y by one third of a right angle. Find the values of x and y.



Sol. We are given

$x = y + 1/3(90º)$ ...(i)

$\Rightarrow x = y + 30º$

Also, $x + y = 180º$ (Linear Pair Axiom). ...(ii)

Putting : $x = y + 30º$ in (ii) we get

$y + 30º + y = 180º$

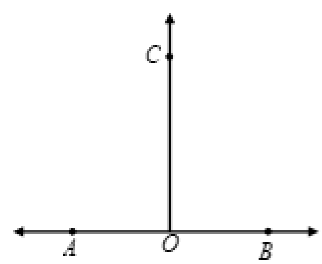
$2y = 150º$

$y = 150/2 = 75º$

From (i) , $x = 75 + 30º = 105º$

Ex.2 If ray OC stands on line AB such that $\angle AOC = \angle COB$, then show that $\angle AOC = 90º$.

Sol. Since ray OC stands on line AB. Therefore,



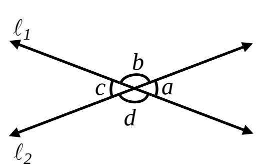
$\angle AOC + \angle COB = 180º$ [Linear pair] ...(i)

But $\angle AOC = \angle COB$ (Given)

$\therefore \angle AOC + \angle AOC = 180º$

$\Rightarrow 2\angle AOC = 180º \Rightarrow \angle AOC = 90º$

Ex.3 In Fig., lines λ_1 and λ_2 intesect at O, forming angles as shown in the figure. If a = 35º, find the values of b, c, and d.



Sol. Since lines λ_1 and λ_2 intersect at O. Therefore,

$\angle a = \angle c$ [Vertically opposite angles]

$\Rightarrow \angle c = 35º$ [$\therefore \angle a = 35º$]

Clearly, $\angle a + \angle b = 180º$

[Since $\angle a$ and $\angle b$ are angles of a linear pair]

$\Rightarrow$ $35^\circ + \angle b = 180^\circ$

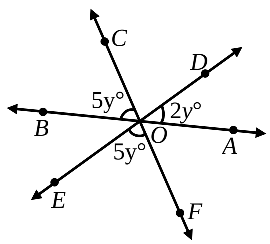
$\Rightarrow$ $\angle b = 180^\circ - 35^\circ$

$\Rightarrow$ $\angle b = 145^\circ$

Since $\angle b$ and $\angle d$ are vertically opposite angles. Therefore,

$\angle d = \angle b \Rightarrow \angle d = 145^\circ$ [$\therefore$ $\angle b = 145^\circ$]

Ex.4 In Fig., determine the the value of y.



Sol. Since $\angle COD$ and $\angle EOF$ are vertically opposite angles. Therefore,

$\angle COD = \angle EOF \Rightarrow \angle COD = 5y^\circ$

[$\therefore$ $\angle EOF = 5y^\circ$ (Given)]

Now, OA and OB are opposite rays.

$\therefore$ $\angle AOD + \angle DOC + \angle COB = 180^\circ$

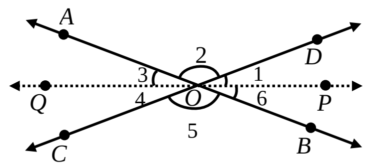
$\Rightarrow$ $2y^\circ + 5y^\circ + 5y^\circ = 180^\circ$

$\Rightarrow$ $12y^\circ = 180^\circ$

$\Rightarrow$ $y^\circ = \dfrac{180}{12} = 15.$

Thus, $y^\circ = 15.$

Ex.5 In Fig., AB and CD are straight lines and OP and OQ are respectively the bisectors of angles BOD and AOC. Show that the rays OP and OQ are in the same line.



Sol. In order to prove that OP and OQ are in the same line, it is sufficient to prove that $\angle POQ = 180º$.

Now, OP is the bisector of $\angle AOC$

$\Rightarrow \angle 1 = \angle 6$ ...(i)

and, OQ is the bisector of $\angle AOC$

$\Rightarrow \angle 3 = \angle 4$ ....(ii)

Clearly, $\angle 2$ and $\angle 5$ are vertically opposite angles.

$\therefore \angle 2 = \angle 5$ ....(iii)

We know that the sum of the angles formed at a point is 360º.

Therefore,

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 = 360º$

$\Rightarrow (\angle 1 + \angle 6) + (\angle 3 + \angle 4) + (\angle 2 + \angle 5) = 360º$

$\Rightarrow 2\angle 1 + 2\angle 3 + 2\angle 2 = 360º$

[Using (i), (ii) and (iii)]

$\Rightarrow 2(\angle 1 + \angle 3 + \angle 2) = 360º$

$\Rightarrow \angle 1 + \angle 2 + \angle 3 = 180º \quad \Rightarrow \angle POQ = 180º$

Hence, OP and OQ are in the same straight line.