INTRODUCTION OF EUCLIDS GEOMETRY

SOME BASIC DEFINITION OF EUCLID GEOMETRY

INTRODUCTION

The word "Geometry" comes from the Greek words "Geo" meaning the 'earth', and 'metrein', meaning 'to measure' Geometry appears to have originated from the need for measuring land. This branch of mathematics was studied in various forms in every ancient civilisation.

Geometry was being developed and applied everywhere in the world. But this was happening in an unsystematic manner. These developments were passed on from one generation to the next, either orally or through palm leaf messages, or by other ways.

The credit for developing the geometry systematically as a science goes to Greeks. Euclid, who is the father of mathematics gave the method of proving geometrical results by deductive reasoning by means of previously proved results. His work is found in thirteen books called '**the elements**'.

- Thales, a Greek mathematician gave '**Basic Proportionality Theorem**'
- **Pythagoras**, one of Thales famous pupils gave the result of '**Pythagoras Theorm**' used in every aspect of mathematics.
- From among the main Indian contributors to geometry were: Bhaskara, who proved Pythagora's Theorm; Aryabhatta, who worked out the area of an isosceles triangle and Brahma Gupta, who discovered the formula for finding the area of cyclic quadrilateral.

Euclid divided the 'Elements' into thirteen chapters, each called a book. Eluclid's Elements form one of the most beautiful and influential works of mathematics. These books influenced the whole world's understanding of geometry. In this chapter, we shall discuss Euclid's approach of geometry and shall try to link it with the present day geometry.

EUCLID'S DEFINITION

Euclid thought geometry as an abstract model of the world in which they lived. The notions of point, lines, plane and so on were derived from what was seen around them. A solid has shape, size, position, and can be moved from one place to another. Its boundaries are called surfaces. They separate one part of the space from another, and are said to have no thickness. The boundaries of the surfaces are curves or straight lines. These lines end in points. We lose one extension called a dimension from solids to points. So a solid has three dimensions, a surface has two a line has one and a point has none. Euclid summarised these statements as definitions.

Definitions :

1. A point is that which has no part.
2. A line is breadthless length.
3. The ends of a line are points.
4. A straight line is a line which lies evenly with the points on itself.
5. A surface is that which has length and breadth only.
6. The edges of a surface are lines.
7. A plane surface is a surface which lies evenly with the straight lines on itself.
8. A plane angle is the inclination to one another of two lines in a plane which meet one another and do not lie in a straight line.
9. When the lines containing the angle are straight, the angle is called rectilinear.
10. When a straight line standing on a straight line makes the adjacent angles equal to one another, each of the equal angles is right, and the straight line standing on the other is called a perpendicular to that on which it stands.
11. An obtuse angle is an angle greater than a right angle.
12. An acute angle is an angle less than a right angle.
13. A boundary is that which is an extremity of anything.
14. A figure is that which is contained by any boundary or boundaries.
15. A circle is a plane figure contained by one line such that all the straight lines falling upon it from one point among those lying within the figure equal one another.

16. The point within the circle from which all the straight lines falling on it are equal is called the centre of the circle.
17. A diameter of the circle is any straight line drawn through the centre and terminated in both directions by the circumference of the circle, and such a straight line also bisects the circle.
18. A semicircle is the figure contained by the diameter and the circumference cut off by it. And the center of the semicircle is the same as that of the circle.
19. Rectilinear figures are those which are contained by straight lines, trilateral figures being those
contained by three, quadrilateral those contained by four, and multilateral those contained by more than four straight lines.
20. Of trilateral figures, an equilateral triangle is that which has its three sides equal, an isosceles triangle is that which has two of its sides alone equal, and a scalene triangle is that which has its three sides unequal.
21. Further, of trilateral figure, a right-angled triangle is that which has a right angle, an obtuse-angled triangle is that which has an obtuse angle, and an acute-angled triangle is that which has its three angles acute.
22. Of quadrilateral figures, a square is that which is both equilateral and right-angled; a rectangle is that which is right-angled but not equilateral; a rhombus is that which is equilateral but not right-angled; and a rhomboid is that which has its opposite sides and angles equal to one another but is neither equilateral nor right-angled. And the quadrilaterals other than these be called trapezia.
23. Parallel straight lines are straight lines which being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction.

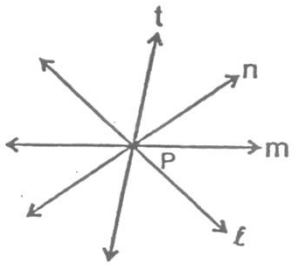
SOME IMPORTANT DEFINATIONS

(i) Collinear points :

Three or more points are said to be collinear if there is a line which contains all of them.

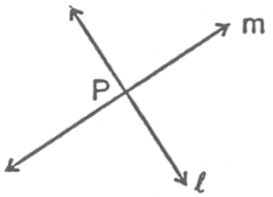
(ii) Concurrent Lines :

Three or more lines are said to be concurrent if there is a point which lies on all of them.



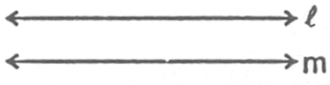
(iii) Intersecting lines :

Two lines are intersecting if they have a common point. The common point is called the "point of intersection".



(iv) Parallel lines :

Two lines *l* and *m* in a plane are said to be parallel lines if they do not have a common point.



(v) Line Segment :

Given two points A and B on a line *l,* the connected part (segment) of the line with end points at A and B, is called the line segment AB.

(vi) Interior point of a line segment :

A point R is called an interior point of a line segment PQ if R lies between P and Q but R is neither P nor Q.

(vii) Congruence of line segment :

Two line segments AB and CD are congruent if trace copy of one can be superposed on the other so as to cover it completely and exactly in this case we write AB≅CD. In other words we can say two lines are congruent if their lengths is same.

(viii) Distance between two points :

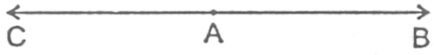
The distance between two points P and Q is the length of line segment PQ

(ix) Ray :

Directed line segment is called a ray. If AB is a ray then it is denoted by $\overrightarrow{AB}$. Point A is called initial point of ray.

(x) Opposite rays :

Two rays AB and AC are said to be opposite rays if they are collinear and point A is the only common point of the two rays.



Ex.1 Give a definition for each of the following terms. Are there other terms that need to be defined first ? What are they, and how might you define them ?

(i) parallel lines (ii) perpendicular lines

(iii) line segment (iv) radius

Sol. **(i) Parallel lines :** Lines which don't intersect any where are called parallel lines.

(ii) Perpendicular lines : Two lines which are at a right angle to each other are called perpendicular lines.

(iii) Line segment : it is a terminated line.

(iv) Radius : The length of the line-segment joining the centre of a circle to any point on its circumference is called its radius.