INTRODUCTION OF EUCLIDS GEOMETRY

AXIOMS AND POSTULATES

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There are certain "obvious universal truths' which are not to be proved but are assumed to be true. Euclid divided these into two types : axioms and postulates.

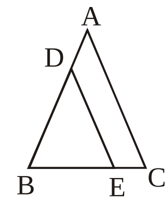
Postulates are universal truths which are specific to geometry, whereas axioms are universal truths used throughout mathematics but not restricted merely to geometry.

Nowadays the words postulate and axiom are used interchangeably :

1. Euclid's Axioms

Some of Euclid's axioms are given below :

1.	Things which are equal to the same thing are equal to one another.	If	$b = a$ and $c = a$ then $b = c$
2.	If equals are added to equals, the wholes are equal	If	$a = b$ and $c = d$ then $a + c = b + d$
3.	If equals are subtracted from equals, the remainders are equal.	If	$a = b$ and $c = d$ then $a - c = b - d$
4.	Things that coincide with one another are equal to one another.		$=$ circles with the same radii



5. The whole is greater than the part.

Area of $\Delta DBE <$ Area of ΔABC

6. Things that are double of the same things are equal to one another. If $y = 2x$ and $z = 2x$, then $y = z$

7. Things that are halves of the same things are equal to one another. If $y = x/2$ and $z = x/2$, then $y = z$

2. Euclid's Five Postulates

Postulate 1:

A straight line may be drawn from any one point to any other point. However, we shall use the following version.

Axiom 1 :

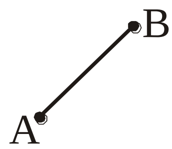
Given any two distinct points in a plane, there exists one and only one line containing them. In other words, two distinct points in a plane determine a unique straight line which contains them.

Through point A there exists infinitely many lines. Similarly through point B, there exists infinitely many lines. But there exists one and only one line through A and B.

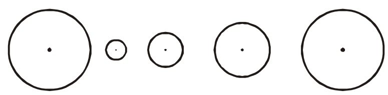
Postulate 2:

A terminated line can be produced indefinitely.

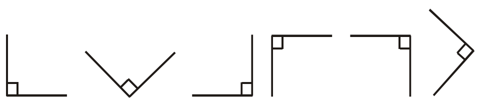
By a terminated line, we mean a line segment. Thus, the second postulate says that a line segment can be extended to either side to form a line.



Postulate 3: A circle can be drawn with any centre and any radius.



Postulate 4 : All right angles are equal to one another.

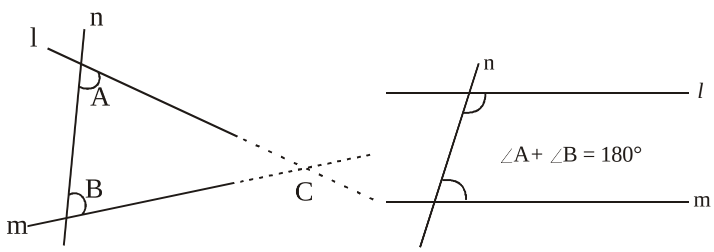


Postulate 5 :

If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely meet on that side on which the sum of the angles is less than two right angles.

If $\angle A + \angle B < 2 \times (90^{\circ}) = 180^{\circ}$, then l and m meet at some point on the side in which $\angle$ A and $\angle$ B lie.

How ever, if $\angle A + \angle B = 180^{\circ}$, then the lines l and m will never meet. In other words. lines l and m will be parallel.



$\angle A + \angle B < 180^{\circ}$, l and m intersect each other $\angle A + \angle B = 180^{\circ}$, l || m

Propositions or Theorems :

These are statements that can be proved by using axioms, definitions, earlier propositions, deductive logic, and by applying reasoning.

Corollary:

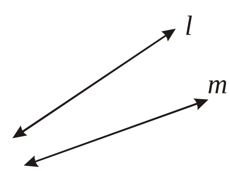
A statement whose truth can be easily derived from a theorem is called its corollary.

Consistent System of Axioms:

A system of axioms is said to be consistent if it is impossible to deduce from these axioms a statement which contradicts any axioms or a statement already proved using these axioms. We now prove the following theorem. We prove it by contradiction.

Theorem 1 :

Two distinct lines cannot have more than one point in common. Given : Two distinct lines l and m.



To prove :

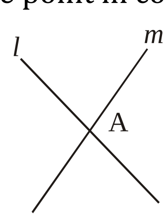
l and m have two distinct points A and B in common. The line l contains the points A and B and also, the line m contains the points A and B. But, according to Axiom 5.1, there is one and only one line which contains two distinct points A and B.

Thus l and m are the same lines.

This contradicts the given hypothesis that l, m are two distinct lines.

Our assumption is wrong.

Hence l and m have at most one point in common.

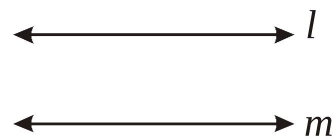


Intersecting lines:

Two lines which have exactly one point in common are said to be intersecting lines, and the common point is called the "**point of intersection**"

Parallel lines:

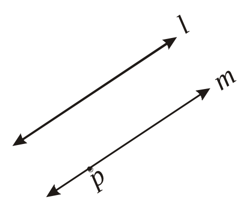
Two lines which have no point in common are said to be parallel lines.



Equivalent Version of Euclid's Fifth Postulate

Axiom 2 : Euclid's Parallel Postulate:

If l is a line and P a point not on l, there is one and only one line which passes through P and parallel to l.



Alternative form of Euclid's Parallel Postulate

Play fair's Axiom:

Two intersecting lines cannot be parallel to the same line.

In other words, given a line l and a point P not on l, there exists one and only one line m through P and parallel to l.

Ex.1 How would you rewrite Euclid' fifth postulate so that it would be easier to understand ?

Sol. Two distinct intersecting lines cannot be parallel to the same line.

Ex.2 Does Euclid's fifth postulate imply the existence of parallel lines ? Explain.

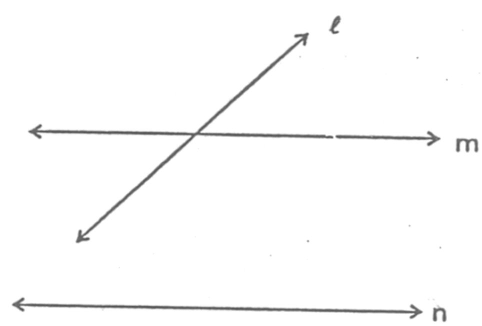
Sol. if a straight line ℓ falls on two straight lines m and n such that sum of the interior angles on one side of ℓ is two right angles, then by Euclid's fifth postulate the line will not meet on this side of ℓ. Next, we know that the sum of the interior angles on the other side of line ℓ also be two right angles. Therefore they will not meet on the other side. So, the lines m and n never meet and are, therefore parallel.

Theorem 1 :

If ℓ, m, n are lines in the same plane such that ℓ intersects m and n $\parallel$ m, then ℓ intersects n also.

Given : Three lines ℓ, m, n in the same plane s.t. ℓ intersects m and n $\parallel$ m.

To prove : Lines ℓ and n are intersecting lines.



Proof : Let ℓ and n be non intersecting lines. Then. $\ell \parallel n$.

But, n $\parallel$ m [Given]

$\therefore \ \ell \parallel n$ and $n \parallel m \Rightarrow \ell \parallel m$

$\Rightarrow$ ℓ and m are non-intersecting lines.

This is a contradiction to the hypothesis that ℓ and m are intersecting lines.

So our supposition is wrong.

Hence, ℓ intersects line n.

Theorem 2 :

If lines AB, AC, AD and AE are parallel to a line ℓ, then points A, B, C, D and E are collinear.

Given : Lines AB, AC, AD and AE are parallel to a line ℓ.

To prove : A, B, C, D, E are collinear.

Proof : Since AB, AC, AD and AE are all parallel to a line ℓ Therefore point A is outside ℓ

and lines AB, AC, AD, AE are drawn through A and each line is parallel to ℓ.

But by parallel lines axiom, one and only one line can be drawn through the point A outside it and parallel to ℓ.

This is possible only when A, B, C, D, and E all lie on the same line. Hence, A, B, C, D and E are collinear.

Ex.3 State which of the following are true statements. Give reasons for your answers.

(a) There exists a point through which no line can pass.

(b) Given two distinct points, infinite number of lines can pass through these points.

(c) If two circles are equal, their radii are equal.

(d) A terminated line can be produced indefinitely on both the sides.

(e) There exists one and only one circle with centre at a given point A.

(f) There exist two numbers x and y such that x y but $2x = 2y$

(g) Two distinct lines can have at most one common point.

Sol.

(a) False. Infinite number of lines can pass through a given point.

(b) False. There exists one and only one line passing through two given points.

(c) True. Circles with different radii cannot be equal.

(d) True: Postulate 2.

(e) False. There exist infinite circles with a given centre.

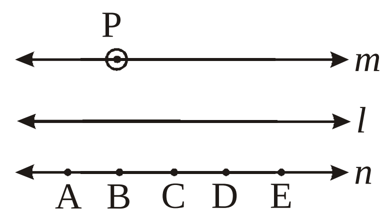
(f) False. Since $2x = 2y \Rightarrow x = y$

(g) True : Theorem 5.1.

Ex.4 If lines AB, AC, AD and AE are parallel to a line l. What can be said about the points A, B, C, D and E ?

Sol.

(i) Since by parallel Playfair's Axiom, if 'l' is a line and a point P not on it, there is one and only one line 'm' which passes through P and is parallel to 'l '



(ii) Here AB, AC, AD and AE are the lines through the point A parallel to the line 'l'. The points A, B,C, D and E all lie on n. Hence, the points A, B, C, D and E are collinear.