COORDINATE GEOMETRY

SECTION FORMULA

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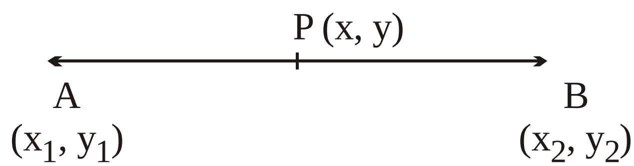
The coordinates of the point P(x, y) which divides the line segment joining the points A(x_1, y_1)and B (x_2, y_2) internally in the ratio m : n are given by.

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$



Note :

(i) If P is the midpoint of AB, then it divides AB in the ratio 1 : 1, so its coordinates are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

(ii) The coordinates of the point which divides the line segment joining the points (x_1, y_1) and (x_2, y_2) externally in the ratio m : n are $\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$.

(iii) If the midpoints of the sides BC, AC and AB of DABC respectively are P(x_1, y_1), Q (x_2, y_2) and R(x_3, x_3), then its vertices are

A ($-x_1 + x_2 + x_3$, $-y_1 + y_2 + y_3$), B ($x_1 - x_2 + x_3$, $y_1 - y_2 + y_3$) and

C ($x_1 + x_2 - x_3$, $y_1 + y_2 - y_3$).

(iv) The fourth vertex of a parallelogram whose three consecutive vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) when taken in order is ($x_1 - x_2 + x_3$, $y_1 - y_2 + y_3$).

Ex.1 Find the mid-point of the line segment joining the points (2, – 6) and (6, – 4) and M be the mid-point of AB.

Sol: Let A (2, – 6) and B (6, – 4) be the given points and M be the mid-point of AB.

Then,

$$M=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{2+6}{2},\frac{-6+(-4)}{2}\right)=(4,-5)$$

Hence, the mid-point of AB is (4, – 5).

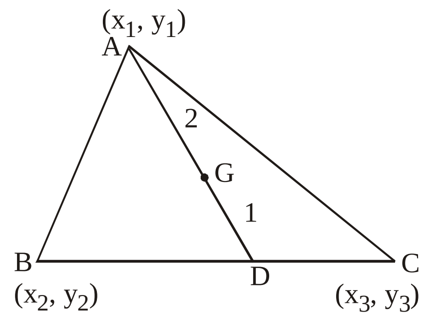
CENTROID OF A TRIANGLE:

Let A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) are vertices of any triangle thenthe centroid is the point of intersection of the medians (Line segment joining the mid-point of a side and its opposite vertex is called a median of the triangle).

Centroid divides the median in the ratio of 2 : 1.

Co-ordinates of centroid G =

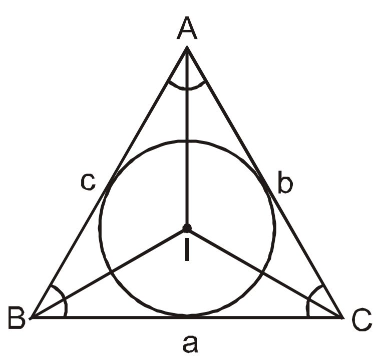
$$\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right).$$



IN-CENTRE OF A TRIANGLE:

The coordinates of the in-centre (intersection point of angle bisector segment) of a triangle whose vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) are

$$\left(\frac{ax_1+bx_2+cx_3}{a+b+c},\frac{ay_1+by_2+cy_3}{a+b+c}\right).$$



Where a, b, c be the lengths of the sides BC, CA, AB respectively.

NOTE :

(i) Incentre divides the angle bisectors in the ratio, (b + c) : a; (c + a) : b & (a + b) : c.

(ii) Orthocenter, Centroid & Circumcenter are always collinear & centroid divides the line joining orthocenter & circumcenter in the ratio 2 : 1 respectively.

(iii) In an isosceles triangle Centrod (G), Orthocenter (O), Incenter (I) & Circumcenter (C) lie on the same line and in an equilateral triangle, all these four points coincide.

Ex.2 Find the coordinates of (i) centroid (ii) in-centre of the triangle whose vertices are (0, 6), (8, 12) and (8, 0).

Sol:(i) We know that the coordinates of the centroid of a triangle whose angular points are (x_1, y_1), (x_2, y_2) (x_3, y_3) are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2,y_3}{3}\right)$

So the coordinates of the centroid of a triangle whose vertices are (0, 6), (8, 12) and (8, 0) are $\left(\frac{0+8+8}{3}, \frac{6+12+0}{3}\right)$ or .

(ii) Let A (0, 6), B (8, 12) and C (8, 0) be the vertices of triangle ABC.

Then $c = AB = \sqrt{(0-8)^2 + (6-12)^2} = 10$

$$b = CA = \sqrt{(0-8)^2 + (6-0)^2} = 10$$

$$a = BC = \sqrt{(8-8)^2 + (12-0)^2} = 12$$

The coordinates of the in-centre are

$$\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$$

$$\left(\frac{12\times0 + 10\times8 + 10\times8}{12+10+10}, \frac{12\times6 + 10\times12 + 10\times0}{12+10+10}\right)$$

$$\left(\frac{160}{32}, \frac{192}{32}\right) = (5,6)$$