COORDINATE GEOMETRY

BASIC INTRODUCTION AND CARTESIAN PLANE

INTRODUCTION:

If we choose x = – 1, 0 etc. from the set R of the real numbers and the corresponding values of y obtained from the relation y = 3x – 2

(i) $x = -1 \Rightarrow y = 3(-1) - 2 = -5$

(ii) $x = 0 \Rightarrow y = 3(0) - 2 = -2$

i.e., –5, –2 are called dependent variables while – 1, 0 are called independent variables. Now the pairs (–1, –5), (0, –2) are known as ordered pairs and coordinates of two different points in the Cartesian plane.

Ordered Pair :

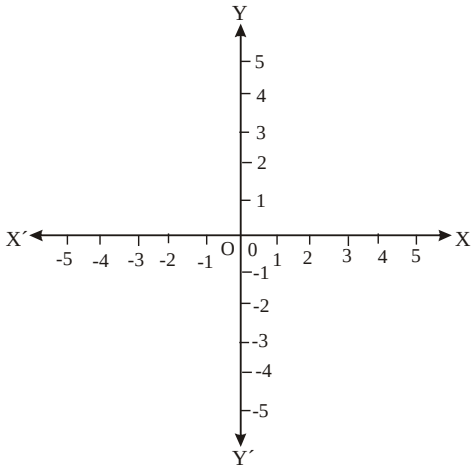
A pair of numbers a and b listed in a specific order with a at the first place and b at the second place is called an ordered pair (a, b).

Note: that (a, b) ^{1} (b, a).

RECTANGULAR COORDINATE AXES:

Let X'OX and Y'OY be two mutually perpendicular lines through any point O in the plane of the paper. This point O, is called **origin**.

Now choose a convenient unit of length and starting from the origin as zero, mark off a number scale on the horizontal line X'OX, positive to the right of the origin O and negative to the left of origin O. Also, mark off the same scale on the vertical line Y'OY, positive upwards and negative downwards of the origin O.



The line X'OX is called the x-axis or axis of x.

The line Y'OY is known as the y-axis or axis of y.

The x-axis and y-axis taken together are called the co-ordinate axis or the axes of coordinates.

Ex.1. Fill in the blanks:

(i) The point in the positive direction to the right of origin at a distance of '4' units from origin lies on _____.

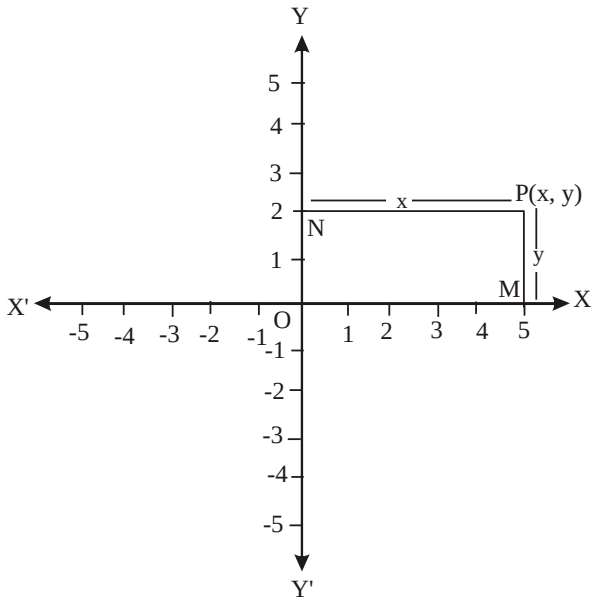
(ii) The point at a distance of 'x' units above the origin lies on _____.

Sol: (i) positive x-axis,

(ii) positive y-axis

CARTESIAN COORDINATES OF A POINT:

Let X'OX and Y'OY be the coordinate axes, and let P be any point in the plane. Draw perpendiculars PM and PN from P on x and y axis respectively. The length of the line segment OM in the units of scale chosen is called the x-coordinate or abscissa of point P. Similarly, the length of the directed line segment ON on the same scale is called the y-coordinate or ordinate of point P. If OM = x and ON = y, then the position of the point P in the plane with respect to the coordinate axes is represented by the ordered pair (x, y). The ordered pair (x, y) is called the coordinates of point P.

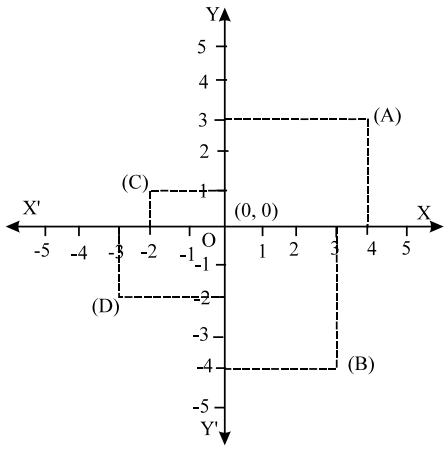


Thus, for a given point, the abscissa and ordinate are the distances of the given point from y-axis and x-axis respectively.

Ex.2. See figure & complete the following statements:

(i) The abscissa and the ordinate of the point A are ___ and ___, respectively. Hence, the coordinates of A are (___,__).

(ii) The abscissa and the ordinate of the point B are ___ and ___, respectively. Hence the coordinates of B are (___, ___).



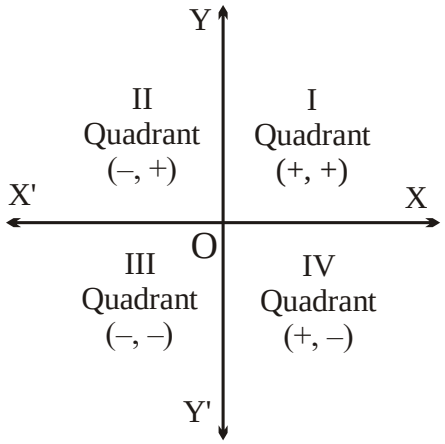
Sol: (i) 4, 3, (4, 3)

(ii) 3, – 4, (3, – 4)

QUADRANTS :

The x-axis and y-axis divide the cartesian plane into four regions, called the quadrants.

Let X'OX and Y'OY be the coordinate axes, which divide the plane in four quadrants. The regions XOY, X'OY and X'OY' and Y'OX are known as the first, the second, the third and the fourth quadrants respectively.



(i) Towards the right side of the Y-axis, x-coordinate of any point on the graph paper is taken positive and towards the left side of the Y-axis, x-coordinate is taken negative.

(ii) Above the X-axis, the y-coordinate of any point on the graph paper is taken positive and below the x-axis, y-coordinate is taken negative.

The four quadrants are characterised by the following signs of abscissa and ordinate:

Quadrant	x coordinate	y coordinate	Point
First Quadrant	+	+	(+, +)
Second Quadrant	–	+	(–, +)
Third Quadrant	–	–	(–, –)
Fourth Quadrant	+	–	(+, –)

NOTE

(i) The coordinates of the origin are (0, 0).

(ii) The coordinates of any point on x axis are of the form (x, 0).

(iii) The coordinates of any point on y axis are of the form (0, y).

(iv) If the abscissa of a point is zero, it would lie on the y axis and if its ordinate is zero it would lie on x-axis.

Ex.3 In which quadrant or on which axis are the points (–4, 6), (5, –2), (–7, 0) and (–2, –1) lie?

Sol: (i) Q x coordinate < 0, y coordinate > 0, point (–4, 6) lies in the II quadrant.

(ii) Q x coordinate > 0, y coordinate < 0, point (5, –2) lies in the IV quadrant.

(iii) Q x coordinate < 0, y coordinate $= 0$, point (–7, 0) lies on x axis.

(iv) Q x coordinate < 0, y coordinate < 0, point (–2, –1) lies in the III quadrant.

Ex.4 Write the coordinates of a point lying on x-axis to the left of origin at a distance of 2 units.

Sol: (–2, 0)

Ex.5 Write the coordinates of a point lying on y-axis at a distance of 5 units above origin.

Sol: (0, 5)

PLOTTING OF POINTS IN THE CARTESIAN PLANE :

In order to plot the points in a plane, we may use the following algorithm:

(i) Draw two mutually perpendicular lines on the graph paper, one horizontal and other vertical.

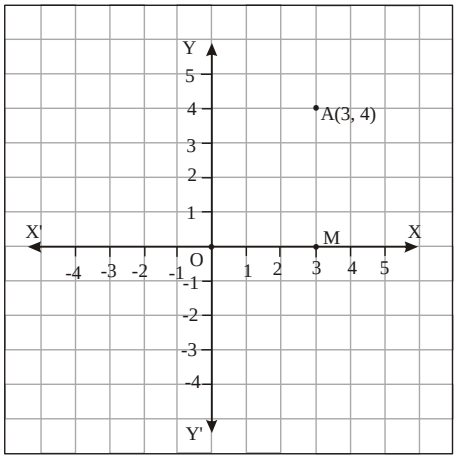
(ii) Mark their intersection point as O (origin). The horizontal line as X'OX and the vertical line as Y'OY. The X'OX is the x-axis and the line Y'OY is the y-axis.

(iii) Choose a suitable scale on x-axis and y-axis and mark the points on both the axes.

(iv) Obtain the coordinates of the point which is to be plotted. Let the point be P(a, b). To plot this point, start from the origin and move '| a |' units along OX or OX' according as 'a' is positive or negative. Suppose we arrive at point M. From point M move vertically upward or downward through | b | units according as b is positive or negative. The point where we arrive finally is the required point P(a, b).

Ex.6 Plot point A(3, 4) on a graph paper.

Sol:



(i) Let X'OX and Y'OY be the coordinate axes.

(ii) Let 1 column on x axis and y axis is equal to 1 units.

(iii) Start from the origin and move 3 units along OX and mark this point as M.

(iv) From M move 4 units vertically upward.

(v) This point is the required point A(3, 4).

Ex.7 Draw the lines X'OX and YOY' as axes on the plane of a paper and plot the points given below:

(i) A(5, 3) (ii) B(–3, 2)

(iii) C(–5, – 4) (iv) D(2, –6)

Sol: Let X'OX and YOY' be the coordinate axes.

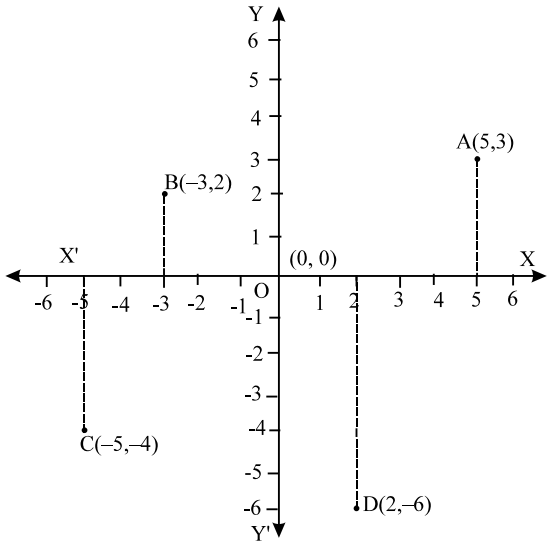
Fix a convenient unit of length and starting from O, mark equal distances on OX, OX', OY and OY':

(i) Starting from O, take +5 units on the x-axis and then +3 units on the y-axis to obtain the point A(5, 3).

(ii) Starting from O, take –3 units on the x-axis and then +2 units on the y-axis to obtain the point B(–3, 2).

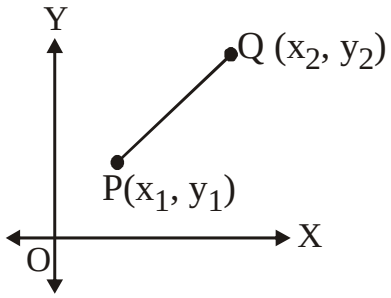
(iii) Starting from O, take –5 units on the x-axis and then –4 units on the y-axis to obtain the point B(–5, –4).

(iv) Starting from O, take 2 units on the x-axis and then –6 units on the y-axis to obtain the point D(2, –6).



DISTANCE BETWEEN TWO POINTS:

The distance between any two points in the plane is the length of the line segment joining them. The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by



$$PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$i.e.\ PQ=\sqrt{(difference\ of\ abscissa)^2+(difference\ of\ ordinates)^2}$$

NOTE

(i) The distance of a point P(x, y) from the origin O(0, 0) is given by OP =

(ii) For three points to be collinear, prove that the sum of the distances between two pairs of points is equal to the third pair of points.

Ex.8 Find the distance between the two points P(–5, 7) and Q(–1, 3).

Sol: $PQ=\sqrt{(-1-(-5))^2+(3-7)^2}$

$\sqrt{(-1+5)^2+(-4)^2}$

$\sqrt{(4)^2+(-4)^2}$

$$PQ = \sqrt{16+16} = \sqrt{32}$$

The distance between two points P and Q is $\sqrt{32}$ units.

APPLICATIONS OF DISTANCE FORMULA:

(i) For given three points A, B, C to decide whether they are collinear or vertices of a particular triangle. First we find the length of AB, BC and CA then we shall find that the points are

(a) Collinear, if the sum of two shorter distances is equal to the largest distance.

(b) Vertices of an equilateral triangle if AB = BC = CA.

(c) Vertices of an isosceles triangle if AB = BC or BC = CA or CA = AB

(d) Vertices of a right angled triangle if $AB^2 + BC^2 = CA^2$ etc.

(ii) For given four points A, B, C and D.

(a) AB = BC = CD = DA; AC = BD ⇒ ABCD is a square.

(b) AB = BC = CD = DA ⇒ ABCD is a rhombus.

(c) AB = CD, BC = DA, AC = BD ⇒ ABCD is a rectangle.

(d) AB = CD, BC = DA ⇒ ABCD is a parallelogram.

(iii) (a) Diagonals of square, rhombus, rectangle and parallelogram always bisect each other.

(b) Diagonals of rhombus and square bisect each other at right angle.

(c) Three given points are collinear if area of the triangle formed from these three points is zero.

(d) Four given points are collinear, if area of quadrilateral formed from these four points is zero.