COORDINATE GEOMETRY

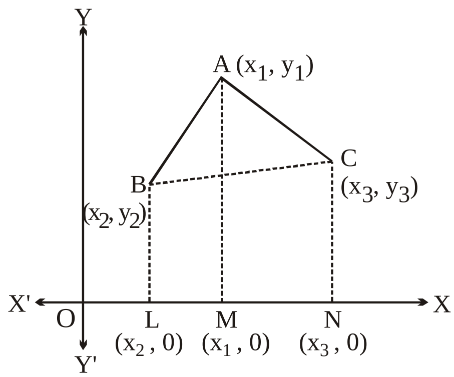
AREA OF TRIANGLE

AREA OF A TRIANGLE :

Let ABC be any triangle whose vertices are A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3). Draw BL, AM and CN perpendiculars from B, A and C respectively on the x-axis. ABLM, AMNC and BLNC are all trapeziums.

Area of DABC = Area of trapezium ABLM + Area of trapezium AMNC – Area of trapezium BLNC

We know that, Area of trapezium $= \frac{1}{2} \times$ (sum of parallel sides) × (distance between them)



Area of $\Delta ABC = \frac{1}{2}(BL + AM)(LM) + \frac{1}{2}(AM + CN)(MN) - \frac{1}{2}(BL + CN)(LN)$

$= \frac{1}{2}(y_2 + y_1)(x_1 - x_2) + \frac{1}{2}(y_1 + y_3)(x_3 - x_1) - \frac{1}{2}(y_2 + y_3)(x_3 - x_2)$

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Ex.1: The vertices of Δ ABC are (–2, 1), (5, 4) and (2, – 3) respectively. Find the area of triangle.

Sol: A (–2, 1), B (5, 4) and C (2, – 3) be the vertices of triangle.

So, $x_1 = -2$, $y_1 = 1$; $x_2 = 5$, $y_2 = 4$; $x_3 = 2$, $y_3 = -3$

Area of Δ ABC $= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

$$\frac{1}{2}\left|\left[(-2)(4+3)+(5)(-3-1)+2(1-4)\right]\right|$$

$$\frac{1}{2}\left|\left[-14+(-20)+(-6)\right]\right|$$

$$\frac{1}{2}\left|-40\right|$$

= 20 Sq. unit.

Ex.2: Find the area of a triangle whose vertices are A(3, 2), B (11, 8) and C(8, 12).

Sol. Let A = (x_1, y_1) = (3, 2), B = (x_2, y_2) = (11, 8) and C = (x_3, y_3) = (8, 12) be the given points. Then,

Area of ΔABC

$= \frac{1}{2} |\{x_1(y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)\}|$

⇒ Area of ΔABC

$=\frac{1}{2} |\{3(8 - 12) + 11 (12 - 2) + 8 (2 - 8)\}|$

⇒ Area of ΔABC

$=\frac{1}{2} |(-12 + 110 - 48)|$ = 25 sq. units

Ex.3: Prove that the area of triangle whose vertices are (t, t – 2), (t + 2, t + 2) and (t + 3, t) is independent of t.

Sol. Let A = (x_1, y_1) = (t, t – 2), B (x_2, y_2) = (t + 2, t + 2) and C = (x_3, y_3) = (t + 3, t) be the vertices of the given triangle. Then,

Area of ΔABC $=\frac{1}{2} | x_1(y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)|$

Area of ΔABC = $\frac{1}{2} |\{t(t + 2 - t) + (t + 2) (t - t + 2) + (t + 3) (t - 2 - t - 2)\}|$

Area of ΔABC $=\frac{1}{2} |\{2t + 2t + 1 - 4t - 12\}| = | - 4 |$

= 4 sq. untis

Clearly, area of ΔABC is independent of t.

Ex.4: Find the area of the triangle formed by joining the mid-point of the sides of the triangle whose vertices are (0, –1), (2, 1) and (0, 3). Find the ratio of area of the triangle formed to the area of the given triangle.

Sol. Let A (0, –1), B(2, 1) and C(0, 3) be the vertices of ΔABC. Let D, E, F be the mid-points of sides BC, CA and AB respectively. Then, the coordinates of D, E and F are (1, 2), (0, 1) and (1, 0) respectively. Now,

Area of ΔABC $=\frac{1}{2}\ |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

Area of ΔABC $=\frac{1}{2}\ |0(1 - 3) + 2(3 - (-1) + 0(0 - 1)|$

Area of ΔABC $= \frac{1}{2}\ |0 + 8 + 0| = 4$ sq. units

Area of ΔDEF $=\frac{1}{2}\ |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

Area of ΔDEF $=\frac{1}{2}\ |1(1 - 0) + 0(0 - 2) + 1(2 - 1)|$

Area of ΔDEF $=\frac{1}{2}\ |1 + 1| = 1$ sq. units

Area of ΔDEF : Area of ΔABC = 1 : 4