POLYNOMIALS

ZEROES OF POLYNOMIAL

ZEROS OF A POLYNOMIAL :

Let p(x) be a polynomial. If p(a) = 0, then we say that a is a zero of the polynomial p(x). The values of x for which the given polynomial vanishes i.e. the value of p(x) becomes zero are called zeros of the polynomial.

For example :

(i) For polynomial $p(x) = x - 2$; $p(2) = 2 - 2 = 0$

$\therefore$ x = 2 or simply 2 is a zero of the polynomial

$p(x) = x - 2$.

(ii) For the polynomial $g(u) = u^2 - 5u + 6$;

$g(3) = (3)^2 - 5 \times 3 + 6 = 9 - 15 + 6 = 0$

$\therefore$ 3 is a zero of the polynomial g(u)

$= u^2 - 5u + 6$.

Also, $g(2) = (2)^2 - 5 \times 2 + 6 = 4 - 10 + 6 = 0$

$\therefore$ 2 is also a zero of the polynomial

$g(u) = u^2 - 5u + 6$

(a) Every linear polynomial has one and only one zero.

(b) A given polynomial may have more than one zeroes.

(c) If the degree of a polynomial is n; the largest number of zeroes it can have is also n.

For example :

If the degree of a polynomial is 5, the polynomial can have at the most 5 zeroes; if the degree of a polynomial is 8; largest number of zeroes it can have is 8.

(d) A zero of a polynomial need not be 0.

For example :

If $f(x) = x^2 - 4$,

then $f(2) = (2)^2 - 4 = 4 - 4 = 0$

Here, zero of the polynomial $f(x) = x^2 - 4$ is 2 which itself is not 0.

(e) 0 may be a zero of a polynomial.

For example :

If $f(x) = x^2 - x$,

then $f(0) = 0^2 - 0 = 0$

Here 0 is the zero of polynomial $f(x) = x^2 - x$.

Ex.1: Verify whether x is a zero of the following polynomials.

(i) $p(x) = 3x + 2, x = -2/3$ (ii) $p(x) = 3x + 1, x = 1/3$

Sol: (i) $p\left(-\frac{2}{3}\right) = 3\left(-\frac{2}{3}\right) + 2$

$= -2 + 2 = 0$

$-2/3$ is a zero of $p(x) = 3x + 2$

(ii) $p\left(\frac{1}{3}\right) = 3\left(\frac{1}{3}\right) + 1 = 1 + 1 = 2 \neq 0$

$1/3$ is not a zero of $p(x) = 3x + 1$

Ex.2: Find the zero of the polynomial in each of the following cases

(i) $p(x) = x + 7$ (ii) $p(x) = ax$, a ¹ 0

Sol: (i) $p(x) = 0$

$\Rightarrow x + 7 = 0$

$\Rightarrow x = -7$

(ii) $p(x) = 0$

$\Rightarrow ax = 0$

$\Rightarrow x = 0/a$

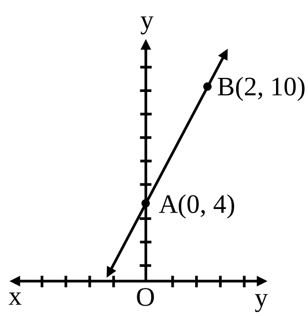
$\Rightarrow x = 0$

GEOMETRIC MEANING OF THE ZEROES OF A POLYNOMIAL :

Let us consider linear polynomial ax + b. The graph of y = ax + b is a straight line.

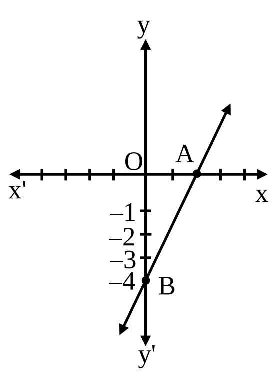
For example : The graph of y = 3x + 4 is a straight line passing through (0, 4) and (2, 10).

x	0	2
y=3x+4	4	10
Points	A	B



(i) Let us consider the graph of y = 2x – 4 intersects the x-axis at x = 2. The zero 2x – 4 is 2. Thus, the zero of the polynomial 2x – 4 is the x-coordinate of the point where the graph y = 2x – 4 intersects the x-axis.

x	2	0
y=2x–4	0	–4
Points	A	B



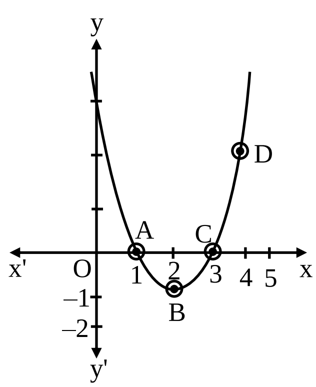
(ii) A general equation of a linear polynomial is ax + b. The graph of y = ax + b is a straight line which intersects the x-axis at $\left(\frac{-b}{a}, 0\right)$.

Zero of the polynomial ax + b is the x-coordinate of the point of intersection of the graph with x-axis.

(iii) Let us consider the quadratic polynomial $x^2 - 4x + 3$. The graph of $x^2 - 4x + 3$ intersects the x-axis at the point (1, 0) and (3, 0). Zeroes of the polynomial $x^2 - 4x + 3$ are the x-coordinates of the points of intersection of the graph with x-axis.

x	1	2	3	4	5
$y=x^2-4x+3$	0	–1	0	3	8
Points	A	B	C	D	E

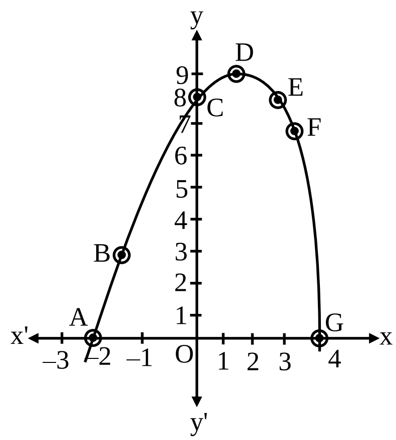
The shape of the graph of the quadratic polynomials is and the curve is known as parabola.



(iv) Now let us consider one more polynomial $-x^2 + 2x + 8$. Graph of this polynomial intersects the x-axis at the points (4, 0) (–2, 0).

Zeroes of the polynomial $-x^2 + 2x + 8$ are the x-coordinates of the points at which the graph intersects the x-axis. The shape of the graph of the given quadratic polynomial is ∩ and the curve is known as parabola.

x	–2	–1	0	1	2	3	4
y	0	5	8	9	8	7	0
Points	A	B	C	D	E	F	G

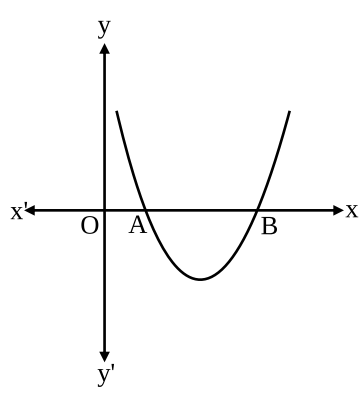


The zeroes of a quadratic polynomial $ax^2 + bx + c$,

$a \neq 0$ are the x-coordinates of the points where the graph of $y = ax^2 + bx + c$ intersects the x-axis. There are three types of the graph of $y = ax^2 + bx + c$.

Case I :

Graph of $y = ax^2 + bx + c$ intersects the x-axis at two distinct points A and B. The zeroes of the quadratic polynomial ax2 + bx + c are the x-coordinates of the points A and B.



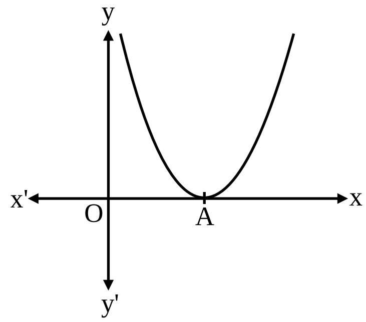
CONDITION : $b^2 - 4ac > 0$ and $a > 0$

For example :

Quadratic polynomial $x^2 - 7x + 12$ Graph of $y = x^2 - 7x + 12$ will cut x-axis at the two distinct points (3, 0) and (4, 0). Zeroes of the polynomial are 3 and 4.

Case II :

Here the graph intersects the x-axis at exactly one point i.e., at two coincident points. These two coincident points A and B coincide and becomes one point A. Zero of the quadratic polynomial is the x-coordinate of point A.



CONDITION : b^2-4ac = 0 and a> 0

For example :

y = $(x - 1)^2$The graph of y = $(x - 1)^2$ will cut x-axis at one point (1, 0). Zero of the polynomial of the point of intersection with x-axis.

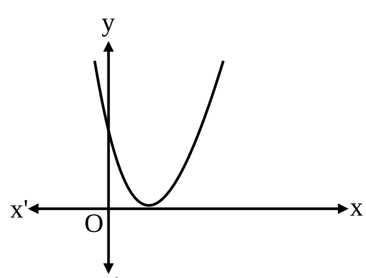
Case III :

Here the graph of the quadratic equation will not cut the x-axis. Either the graph will be completely above the x-axis or below the x-axis So the quadratic polynomial $ax^2 + bx + c$ has no zero in this case.

For example : $y = x^2 - 2x + 4$

Graph of $y = x^2 - 2x + 4$ will not intersect the

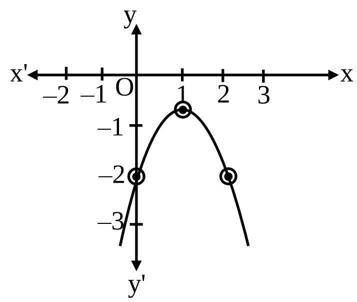
x-axis and the graph will be above the x-axis. The polynomial $x^2 - 2x + 4$ has no zero.



Let $y = -x^2 + 2x - 2$

Graph of $y = -x^2 + 2x - 2$ will not intersect the x-axis and the graph will be below the x-axis.

The polynomial $-x^2 + 2x - 2$ has no zero.



In Brief : It means that a polynomial of degree two has at most two zeroes.

Cubic polynomial :

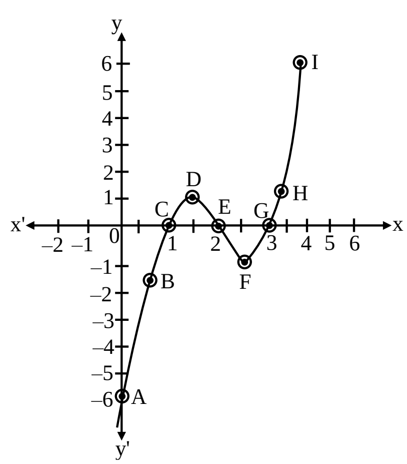
Let us find out geometrically how many zeroes a cubic has.

Let consider cubic polynomial $x^3 - 6x^2 + 11x - 6$.

x	0	0.5	1	1.5	2	2.5	3	3.5	4
$y=x^3-6x^2+11x-6$	–6	–1.875	0	0.375	0	–0.375	0	1.875	6
Points	A	B	C	D	E	F	G	H	I

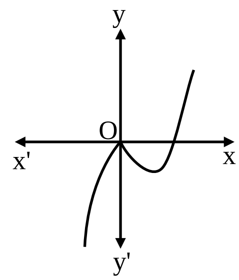
Case 1 :

The graph of the cubic equation intersects the x-axis at three points (1, 0), (2, 0) and (3, 0). Zeroes of the given polynomial are the x-coordinates of the points of intersection with the x-axis.



Case 2 :

The cubic equation $x^3 - x^2$ intersects the x-axis at the point (0, 0) and (1, 0). Zero of a polynomial $x^3 - x^2$ are the x-coordinates of the point where the graph cuts the x-axis.

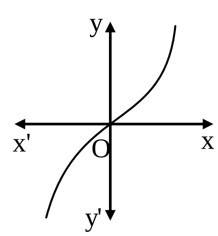


Zeroes of the cubic polynomial are 0 and 1.

Case 3 :

$y = x^3$

Cubic polynomial has only one zero.



In brief : A cubic equation can have 1 or 2 or 3 zeroes or any polynomial of degree three can have at most three zeroes.

Remarks : In general, polynomial of degree n, the graph of y = p(x) passes x-axis at most at n points. Therefore, a polynomial p(x) of degree n has at most n zeroes.