POLYNOMIALS

REMAINDER THEOREM

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Let f(x) be a polynomial of degree n^{3} 1 and let a be any real number. When f(x) is divided by (x–a), then the remainder is f(a).

Proof :

Let p(x) be any polynomial with degree greater than or equal to 1. Suppose that when p(x) is divided by x – a, the quotient is q(x) and the remainder is r(x), i.e., p(x) = (x – a) q(x) + r(x)

Since the degree of x – a is 1 and the degree of r(x) is less than the degree of x – a, the degree of r(x) = 0. This means that r(x) is a constant, say r.

So, for every value of x, r(x) = r.

Therefore, p(x) = (x – a) q(x) + r

In particular, if x = a, this equation gives us

p(a) = (a – a) q(a) + r

= r,

which proves the theorem.

Ex.1 : Find the remainder when $p(x) = 4x^3 - 3x^2 + 2x - 4$ is divided by (x – 1)

Sol: By remainder theorem, we know that when p(x) is divided by x–1, then remainder is p(1).

Now, $p(1) = 4(1)^3 - 3(1)^2 + 2(1) - 4 = 4 - 3 + 2 - 4 = -1$

Hence, the required remainder is –1.

Ex.2: Find the remainder when the polynomial $f(x) = x^3 - 3x^2 + 4x + 50$ is divided by(x+ 3)

Sol: By the remainder theorem, we know that when f(x) is divided by (x + 3), the remainder is f(–3).

Now, $f(-3) = [(-3)^3 - 3 \times (-3)^2 + 4 \times (-3) + 50]$

$= [-27 - 27 - 12 + 50] = -16$

Hence, the required remainder is −16.