POLYNOMIALS

INTRODUCTION OF POLYNOMIALS

INTRODUCTION :

In our previous classes, we have learnt about algebraic expressions and various operations on them. In this chapter, we shall review these concepts and extend them to particular types of expressions, known as polynomials.

We shall come across two types of symbols, namely, constants and variables, defined below:

Constants:

A symbol having a fixed numerical value is called a constant.

For example, 8, –6, 5/7, p etc are all constants

Variables:

A symbol which may be assinged different numerical values is known as a variable.

For example, circumference of a circle is given by

c = 2pr

Here, 2 and p are constants, while c and r are variables.

Algebraic Expressions:

A combination of constants and variables, connected by operations +, –, × and ¸ is known as an algebraic expressions.

Terms of an Algebraic Expression:

The several parts of an algebraic expression separated by

+ or – operations are called the terms of the expression.

For example : $4+9x-5x^2y+\frac{3}{5}xy$ is an algebraic expression containing four terms, namely,

4, 9x, $-5x^2y$ and 3/5 xy.

Coefficient of a term :

Consider a algebraic expression $3x^2 + 5x + 6$. In $3x^2 + 5x + 6$, $3x^2$ is first term, $5x$ is second term and 6 is the third term. In the first term $3x^2$, 3 is called numerical coefficient and x^2 is called literal coefficient. Similarly in the second term $5x$, 5 is called numerical coefficient and x is called literal coefficient.

Like terms :

In any algebraic expression, the terms having the same literal coefficients are called like terms. For example : $6x^3$, $-x^3$, $2x^3$ and $\frac{1}{4}x^3$ are like terms.

Unlike terms :

In any algebraic expression, the terms having different literal coefficient are called unlike terms.

For example : $7x$, x^2, $2x^3$ and $15x^4$ are unlike terms.

POLYNOMIALS :

An algebraic expression f(x) of the form $f(x) = a_0 + a_1x + a_2x^2 + \text{.........} + a_nx^n$; where a_0, a_1, a_2,........,a_n are real numbers and all the indices of variable x are non-negative integers, is called a polynomial in variable x and the highest indices n is called the degree of the polynomial, if a_n ^{1} 0. Here, a_0, a_1x, a_2x^2, and a_nx^n are called the terms of the polynomial and a_0, a_1,a_2...............a_n are called various coefficients of the polynomial f(x). A polynomial in x is said to be in standard form when the terms are written either in increasing order or in decreasing order of the indices of x in various terms.

For example :

$$x^2 - a^2,\ ax^2 + bx + c, x^3 + 3x^2 + 3x + 1,\ y^3 - 7y + 6$$

etc. are the polynomials written in their standard form.

Ex.1: Which of the following expressions are polynomials?

(i) $\dfrac{1}{x^{-2}} + \dfrac{1}{x^{-1}} + \dfrac{1}{2}$

(ii) $\dfrac{1}{x}(x-1)(x-2)$

(iii) $\dfrac{(x^2+x+1)(x+1)}{(1+x)}$

(iv) $x^2 + \dfrac{1}{x^2}$

Sol: (i) $\frac{1}{x^{-2}}+\frac{1}{x^{-1}}+\frac{1}{2}$

$= x^2 + x^1 + \frac{1}{2} \ x^0$ [It is a polynomial]

(ii) $\frac{1}{x}(x-1)(x-2) = x^{-1}\ (x^1 - 1)\ (x^1 - 2)$

$= x^{-1}\ (x^2 - 3x + 2)$

$= x - 3 + 2x^{-1}$

$= x^1 - 3x^0 + 2x^{-1}$ [Its not a polynomial]

(iii) $\frac{(x^2+x+1)(x+1)}{(1+x)}$

$= x^2 + x + 1 = x^2 + x^1 + 1 \cdot x^0$ [It is a polynomial]

(iv) $x^2+\frac{1}{x^2}$ $= x^2 + x^{-2}$ [It is not a polynomial]

Ex.2 : Rewrite the following polynomials in the standard form :

(i) $x - 7 + 8x^2 + 9x^3$

(ii) $-5x^2 + 6 - 3x^3 + 4x$

Sol: (i) $x - 7 + 8x^2 + 9x^3 = 9x^3 + 8x^2 + x - 7$

(ii) $- 5x^2 + 6 - 3x^3 + 4x = - 3x^3 - 5x^2 + 4x + 6$.

Degree of a Polynomial in One Variable :

In case of a polynomial in one variable, the highest power of the variable is called the degree of the polynomial.

Degree of a Polynomial in Two or More Variables :

In case of polynomials in more than one variable, the sum of the powers of the variables in each term is taken up and the highest sum so obtained is called the degree of the polynomial.

Ex.3 : Write the degree of each of the following polynomials.

(i) $5x^3 + 4x^2 + 7x$

(ii) $4 - y^2$

(iii) $5t - \sqrt{7}$

(iv) 3

(v) $7x^3 - 5x^2y^2 + 3xy + 6y + 8$

Sol: (i) In the given polynomial $5x^3 + 4x^2 + 7x$, the highest degree of variable x is 3.

(ii) In the given polynomial $4 - y^2$, the highest degree of variable y is 2.

(iii) In the given polynomial $5t - \sqrt{7}$, the highest degree of variable t is 1.

(iv) In the given constant polynomial 3 i.e. $3x^\circ$, the highest degree of any variable like x is 0 (zero).

(v) The given polynomial is in 2 variables x and y. The highest sum of the powers of the x and y in each term is 4. Therefore, degree of polynomial is 4.

TYPES OF POLYNOMIALS :

Polynomials can be classified on the basis of number of terms and on the basis of degree.

On the basis of degree :

(i) Constant polynomial :

A polynomial of degree zero is called a constant polynomial.

For example : 2, –5, 7. Every real number is a constant polynomial

(ii) Linear polynomial :

A polynomial of degree 1 is called a linear polynomial.

For example : $3x + 5$ is a linear polynomial in x.

$x + y + 8$ is a linear polynomial in x and y.

(iii) Quadratic polynomial :

A polynomial of degree 2 is called a quadratic polynomial.

For example : $3y^2 - 8y + 5$ is a quadratic polynomial in y.

$2xy + 5x + 3y + 4$ is a quadratic polynomial in x and y.

(iv) Cubic polynomial :

A polynomial of degree 3 is called a cubic polynomial.

For example : $4x^3 - 3x^2 + 7x + 1$ is a cubic polynomial in x

$4x^2y + 5xy^2 + 8$ is a cubic polynomial in x and y.

(v) Biquadratic polynomial :

A polynomial of degree 4 is called a biquadratic polynomial.

For example : $z^4 + 6z^3 + 10z^2 + 6z + 1$ is biquadratic polynomial in z.

$3x^2yz + 4xy^2z + 5xyz^2$ is biquadratic polynomial in, x, y and z.

Ex.4 : Classify the following as linear, quadratic and cubic polynomials.

(i) $x^2 + x$ (ii) $x - x^3$

(iii) $y + y^2 + 4$ (iv) $1 + x$

(v) $3t$ (vi) r^2

(vii) $7x^3$

Sol: (i) $x^2 + x$, it is quadratic polynomial as the highest degree of variable x is 2.

(ii) $x - x^3$, it is cubic polynomial as the highest degree of variable x is 3.

(iii) $y + y^2 + 4$, it is a quadratic polynomial as the highest degree of variable y is 2.

(iv) $1+x$, it is a linear polynomial as the highest degree of x is 1.

(v) $3t$, it is a linear polynomial as the highest degree of t is 1.

(vi) r^2, it is a quadratic polynomial as the highest degree of r is 2.

(vii) $7x^3$, it is a cubic polynomial as the highest degree of x is 3.

On the basis of Number of terms :

(i) Zero polynomial:

A polynomial consisting of one term namely zero only is called a zero polynomial.

Note : "The degree of zero polynomial is not defined."

For example, '0' is called a zero polynomial :

(ii) Monomial:

A polynomial containing one non zero term is called as monomial.

(iii) Binomial:

A polynomial containing two non zero terms is called as binomial.

(iv) Trinomial:

A polynomial containing three non-zero terms is called as trinomial.

Note : "No specific name is given to those polynomials which have more than three terms".

Value of polynomial :

For a polynomial $f(x) = 3x^2 - 4x + 2$. To find its value at $x = 3$;

replace x by 3 everywhere.

So, the value of $f(x) = 3x^2 - 4x + 2$ at $x = 3$ is

$f(3) = 3 \times 3^2 - 4 \times 3 + 2$

$= 27 - 12 + 2 = 17.$

Similarly, the value of polynomial

$f(x) = 3x^2 - 4x + 2,$

(i) at $x = -2$ is $f(-2) = 3(-2)^2 - 4(-2) + 2$

$= 12 + 8 + 2 = 22$

(ii) at $x = 0$ is $f(0) = 3(0)^2 - 4(0) + 2$

$= 0 - 0 + 2 = 2$

(iii) at $x = \frac{1}{2}$ is $f\left(\frac{1}{2}\right) = 3\left(\frac{1}{2}\right)^2 - 4\left(\frac{1}{2}\right) + 2$

$= \frac{3}{4} - 2 + 2 = \frac{3}{4}$