PROBABILITY

INTRODUCTION OF PROBABILITY

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In our day-to-day life we generally use the words probably or probable or change (s) or likely etc. In many statements such as :

(i) It will probably rain today.

(ii) He is probably right

(iii) India's cricket team has good chances of winning the world cup 2011.

In such statements, we generally use the words. 'probably', 'chances' most probably etc. These words convey the sense that the event is not certain to take place or, in other words, there is uncertainty about the occurrence (or happening) of the event in the question. In this chapter, we shall introduce the concept of probability as a measure of uncertainty. Probability is a concept which numerically measure the degree of uncertainty and, therefore, of certainty of the occurrence of an event.

EXPERIMENT

An operation which can produce some well-defined outcomes is known as an experiment.

Random Experiment

If in each trial of an experiment, conducted under identical conditions, the outcomes in not unique, but may be any of the several possible outcome is then such an experiment is known as random experiment.

In a random experiment, the outcome of each trial depends on chance.

Tossing a fair coin, rolling an unbiased die, drawing a card from a well-shuffled pack of cards are all examples of random experiments.

SAMPLE SPACE

The set of all possible outcomes in a random experiment is called a sample space and it is generally denoted by S.

Each element of a sample space is called a sample point.

For Example

A coin is tossed twice. If the second throw results in a tail then a die is thrown.

Then

Clearly, the sample space is given by

S = {HH, TH, HT1, HT2, HT3, HT4, HT5, HT6, TT1, TT2, TT3, TT4, TT5, TT6}

For Example

An experiment consists of rolling a die and then tossing a coin once if the number on the die is even. If the number on the die is odd, the coin is tossed twice.

Then

Clearly, the sample space is given by

S= {2H, 2T, 4H, 4T, 6H, 6T, 1HH, 1HT, 1TH, 1TT, 3HH, 3HT, 3TH, 3TT, 5HH, 5HT, 5TH, 5TT}

EVENT

Every subset of a sample space is called an event.

For Example

In a single throw of a die, we have

Sample space S = {1, 2, 3, 4, 5, 6}.

The event of getting a prime number is given by

E = {2, 3, 5}

Clearly E S.

1 Impossible Event

Let S be a sample space.

Since ϕ S, ϕ is an event, called an impossible event.

In throwing a dice, there are only six possible outcomes 1, 2, 3, 4, 5 and 6 . Let we are interested in getting a number 7 on throwing a dice. Since no face of the dice is marked with 7, so 7 cannot come under any circumstances. Hence getting a 7 is impossible, this type of event is called an impossible event.

$P(7) = \frac{0}{6} = 0$

Probability of an impossible event is always zero.

2 Sure Event

Let S be a sample space.

Since S S, S is an event, called a sure event.

For Example

In a throw of a die, we have

sample space S = {1, 2, 3, 4, 5, 6}.

Let E_1 = event of getting a number less than 1.

And, E_2 = event of getting a number less than 7.

Clearly, no outcome can be less then 1.

E_1 is an impossible event.

Also, each outcome is a number less than 7.

E_2 is a sure event.

3 Simple Event

An event containing only a single element of the sample space is called a simple or an elementary event.

4 Compound Event

An event which is not simple is called a **compound** or **composite** or **mixed event.**

For Example

In a simultaneous toss of two coins, we have

sample space S = {HH, HT, TH, TT}.

Then. E_1 = event of getting a tail on the coins = {TT}, is a simple event.

And, E_2 = event of getting at least 1 tail = {HT, TH, TT}, is a compound event.

MUTUALLY EXCLUSIVE EVENTS

Two events E_1 and E_2 are said to be mutually exclusive if $E_1 \ E_2 = \phi$.

However, if $E_1 \ E_2 \neq \phi$ then E_1 and E_2 are called compatible events.

For Example

(i) In throwing a die, we have S = {1, 2, 3, 4, 5, 6}.

Let E_1 = event of getting a number less than 3.

And, E_2 = event of getting a number more than 4.

Then, $E_1 = \{1, 2\}$ and $E_2 \ \{5, 6\}$.

Clearly, $E_1 \ E_2 = \phi$.

Hence, E_1 and E_2 are mutually exclusive.

Let E_1 = event of getting a head on the first coin = {HH, HT}

And, E_2 = event of getting a tail on the second coin = {HT, TT}.

Clearly, $E_1 \ E_2 \neq \phi$.

Hence, E_1 and E_2 are compatible events.

Ex.1 Two dice are rolled. Let A, B, C be the events of getting a sum of 2, a sum of 3 and a sum of 4 respectively. Then, show that

(i) A is a simple event

(ii) B and C are compound events

(iii) A and B are mutually exclusive

Sol. Clearly, we have

A = {(1, 1)}, B = {(1, 2), (2, 1)}, and C = {(1, 3), (3, 1), (2, 2)}.

(i) since A consists of a single sample point, it is a simple event.

(ii) since Both B and C contain more than one sample point, each one of them is a compound event.

(iii) since $A \ B = \phi$, A and B are mutually exclusive.

Ex.2 From a group of 2 boys and 3 girls, two children are selected at random. Describe the events.

(i) A = event that both the selected children are girls.

(ii) B = event that the selected group consists of one boy and one girl.

(iii) C = event that at least one boy is selected which pairs of events are mutually exclusive?

Sol Let us name the boys as B_1 and B_2, and the girls as G_1, G_2 and G_3. Then,

$S = \{B_1B_2, B_1G_1, B_1G_2, B_1G_3, B_2G_1, B_2G_2, B_2G_3, G_1G_2, G_1G_3, G_2G_3\}$.

We have

(i) $A = \{G_1G_2, G_1G_3, G_2G_3\}$

(ii) $B = \{B_1G_1, B_1G_2, B_1G_3, B_2G_1, B_2G_2, B_2G_3\}$

(iii) $C = \{B_1G_1, B_1G_2, B_1G_3, B_2G_1, B_2G_2, B_2G_3, B_1B_2\}$

Clearly, $A \quad B = \phi$ and $A \quad C = \phi$.

Hence, (A, B) and (A, C) are mutually exclusive events.

PROBABILITY

Theory of probability deals with measurement of uncertainty of the occurrence of same event or incident in terms of percentage or ratio.

(i) Sample Space : Set of possible out comes.

(ii) Trial : Trial is an action which results in one of several outcomes.

(iii) An experiment :

An experiment is any kind of activity such as throwing a die, tossing a coin, drawing a card. outcome of an experiment. The different possibilities which can occur during an experiment.

e.g. on throwing a dice, 1 dot, 2 dots, 3 dots, 4 dots, 5 dots, 6 dots can occur.

(iv) An event : getting a ‘six’, in a throw of dice, getting a head, in a toss of a coin.

(v) A random experiment : Whenever we do some experiment at once.

(vi) Equally likely outcomes :

there are equal uncertainty in getting 1 dot, 2 dots, 3 dots, 4 dots, 5 dots, 6 dots when we throw a single dice.

(vii) Probability of an event A:

Written as P(A) in a random experiment and is defined as -

$$P(A) = \frac{\text{Number of outcomes in favour of A}}{\text{Total number of possible outcomes}}$$

(a) Important Properties :

(i) $0 \le P(A) \le 1$

(ii) P (not happening of (A) + P(happening of A) = 1

or $P(\overline{A}) = P(A) = 1$

$\therefore$ $P(\overline{A}) = 1 - P(A)$

Probability of the happening of A = $\frac{\text{Number of favourable outcomes}}{\text{Total number possible outcomes}} = \frac{m}{m+n}$

Probability of not happening of A (falling of A) = $\frac{n}{m+n}$

where is for an event A can happen in m ways and fail in n ways all these ways being equally likely to occur.

(b) Problems of Die :

(i) A die is thrown once. What is the probability of -

(A) Getting an even number in the throwing of a die, the total number of outcomes is 6.

Let be the event of getting an even number then there are three even numbers 2, 4, 6.

$\therefore$ number of favourable outcomes = 3.

$\therefore$ $P(A) = \frac{\text{no of faourable outcomes}}{\text{total no of outcomes}} = \frac{3}{6} = \frac{1}{2}.$

(B) Getting an odd number (A) total outcomes = 6, favourable outcomes = 3(1, 3, 5)

$\therefore \quad P(A) = \frac{3}{6} = \frac{1}{2}$

(C) Getting a natural number P(A) $= \frac{6}{6} = 1$

(D) Getting a number which is multiple of 2 and 3 $= \left(\frac{\text{Fabourable cases}}{6}\right)$

(E) Getting a number $\geq 3 (3,4,5,6)$

$P(A) = \frac{4}{6} = \frac{2}{3}$

(F) Getting a number 5 or 6 (5 or 6) P(A) = $\frac{2}{6} = \frac{1}{3}$

(G) Getting a number ≤ 5 P(A) = $\frac{5}{6}$ (1,2,3,4,5)

(c) Problems Concerning Drawing a Card :

(i) A pack of 52 cards

(ii) Face cards (King, Queen, Jack)

Ex.3 A card is drawn from a well shuffled deck of 52 cards. Find the probability of

(i) A king.

(ii) A heart.

(iii) A seven of heart.

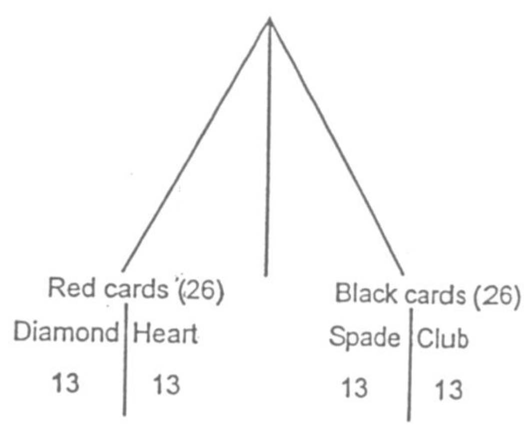
(iv) A jack, queen or a king.

(v) A two of heart or a two of diamond.

(vi) A face card.

(vii) A black card.

(viii) Neither a heart nor a king.



(ix) Neither an ace nor a king.

Sol. Total no. of outcomes = 52

(i) A king.

No. of kings = 4 (favorable cases) $P(A)=\frac{4}{42}=\frac{1}{13}$.

(ii) A heart $P(A)=\frac{13}{52}=\frac{1}{4}$

(iii) A seven of heart $P(A)=\frac{1}{52}$

(iv) A jack, queen or a king $P(A)=\frac{12}{52}=\frac{3}{13}$

(v) A two of heat or a two of diamond. $P(A)=\frac{2}{52}=\frac{1}{26}$

(vi) A face card $P(A)=\frac{12}{52}=\frac{3}{13}$

(vii) A black card $P(A)=\frac{26}{52}=\frac{1}{2}$

(viii) Neither a heart nor a king (13 heart + 4 king, but 1 common)

$$P(A)=1-\frac{16}{52}=\frac{52-16}{52}=\frac{36}{52}=\frac{9}{13}$$

(ix) Neither an ace nor a king. $P(A)=\frac{44}{52}=\frac{11}{13}$

Ex.4 Two coins are tossed simultaneously. Find the probability of getting

(i) two heads

(ii) at least one head

(iii) no head

Sol. $\therefore$ On tossing two coins simultaneously, all the possible outcomes are

HH, HT, TH, TT.

(i) The probability of getting two heads = P (HH)

$$= \frac{\text{Event of occurence of two heads}}{\text{Total number of possible outcomes}} = \frac{1}{4}$$

(ii) The probability of getting at least on head

$$= \frac{\text{Favourable outcomes}}{\text{Total no of outcomes}} = \frac{3}{4}$$

(iii) The probability of getting no head P(TT) = $\frac{1}{4}$

Ex.5 A bag contains 5 red balls, 8 white balls, 4 green balls and 7 black balls. If one ball is drawn at random, find the probability that it is

(i) Black

(ii) Not red

(iii) Green

Sol. Number of red balls in the bag = 5

Number of white balls in the bag = 8

Number of green balls in the bag = 4

Number of black balls in the bag = 7

$\therefore$ Total number of balls in the bag = 5 + 8 + 4 + 7 = 24.

Drawing balls randomly are equally likely outcomes.

$\therefore$ Total number of possible outcomes = 24

Now,

(i) There are 7 black balls, hence the number of such favourable outcomes = 7

$\therefore$ Probability of drawing a black ball = $\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}} = \frac{7}{24}$ **Ans.**

(ii) There are 5 red balls, hence the number of such favourable outcomes = 5.

$\therefore$ Probability of drawing a red ball = $\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcoes}} = \frac{5}{24}$

$\therefore$ Probability of drawing not a red ball = P (Not Red ball) = $1 - \frac{5}{24} = \frac{19}{24}$

(iii) There are 4 green balls.

$\therefore$ Number of such favourable outcomes = 4

Probability of drawing a green ball = $\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}} = \frac{4}{24} = \frac{1}{6}$ **Ans.**

Ex.6 A card is drawn from a well - shuffled deck of playing cards. Find the probability of drawing

(i) a face card

(ii) a red face card

Sol. Random drawing of cards ensures equally likely outcomes

(i) Number of face cards (King, Queen and jack of each suits) = $4 \times 3 = 12$

Total number of cards in deck = 52

$\therefore$ Total number of possible outcomes = 52

P (drawing a face card) = $\frac{12}{52} = \frac{3}{13}$

(ii) Number of red face cards = $2 \times 3 = 6$

Number of favourable outcomes of drawing red face card = 6

P (drawing of red face red) $= \frac{6}{52} = \frac{3}{26}$ **Ans.**

Ex.7 The record of a weather station shows that out of the past 250 consecutive days, its weather forecasts were correct 175 times.

(i) What is the probability that on a given day it was correct ?

(ii) What is the probability that it was not correct on a given day ?

Sol. The total number of days for which the record is available = 250

(i) P(correct forecast)

$$= \frac{\text{Number of days when the forecast was correct}}{\text{Total number of days for which the record is available}} = \frac{175}{250} = 0.7$$

(ii) The number of days when the forecast was not correct = 250 – 175 = 75.

P(not correct forecast) = $\frac{75}{250} = 0.3$

Ex.8 If the probability of winning a game is 0.3, what is the probability of lossing it ?

Sol. Probability of winning a game = 0.3.

Probability of losing it = q (say).

$\Rightarrow$ 0.3 + q = 1

$\Rightarrow$ q = 1 – 0.3

$\Rightarrow$ q = 0.7

Ex.9 Two coins are tossed simulataneously. Find the probability of getting

(i) two heads

(ii) at least one head

(iii) no head

Sol. Let H denotes head and T denotes tail.

$\therefore$ On tossing two coins simultaneously, all the possible outcomes are

(i) The probability of getting two heads = P(HH)

$$= \frac{\text{Event of occurence of two heads}}{\text{Total number of possible outcomes}} = \frac{1}{4}$$

(ii) The probability of getting at least one head

= P(HT or TH or HH)

$$= \frac{\text{Event of occurence of atleast one head}}{\text{Total number of possible outcomes}} = \frac{3}{4}$$

(iii) The probability of getting no head = P(TT)

$$= \frac{\text{Event of occurence of no head}}{\text{Total number of possible outcomes}} = \frac{1}{4}$$

Ex.10 On tossing three coins at a time, find -

(i) All possible outcomes.

(ii) events of occurence of 3 heads, 2 heads, 1 head and 0 head.

(iii) probabilty of getting 3 heads, 2 heads, 1 head and no head.

Sol. Let H denotes head and T denotes tail. On tossing three coins at a time,

(i) All possible outcomes = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}. These are the 8 possible outcomes.

(ii) An event of occurence of 3 heads = (HHH) = 1

An event of occurence of 2 heads = {HHT, HTH, THH} = 3

An event of occurence of 1 head = {HTT, THT, TTH} = 3

An event of occurence of 0 head = {TTT} = 1

(iii) Now, probability of getting 3 heads = P (HHH)

$$= \frac{\text{Event of occurence of 3 heads}}{\text{Total number of possible outcomes}} = \frac{1}{8}$$

Simultaneously, probability of getting 2 heads= P(HHT or THH or HTH)

$$= \frac{\text{Event of occurence of 2 heads}}{\text{Total number of possible outcomes}} = \frac{3}{8}$$

Probability of getting one head = P (HTT or THT or TTH)

$$= \frac{\text{Event of occurence of 1 head}}{\text{Total number of possible outcomes}} = \frac{3}{8}$$

Probability of getting no head = P (TTT)

$= \frac{\text{Event of occurence of no head}}{\text{Total number of possible outcomes}} = \frac{1}{8}$

Ex.11 A bag contains 12 balls out of which x are white,

(i) If one ball is drawn at random, what is the probability that it will be a white ball ?

(ii) If 6 more white balls are put in the bag, the probability of drawing a white ball will double than that in (i). Find x.

Sol. Random drawing of balls ensures equally likely outcomes

Total number of balls = 12

Total number of possible outcomes = 12

Number of white balls = x

(i) Out of total 12 outcomes, favourable outcomes = x

$P(\text{White ball}) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}} = \frac{x}{12}$

(ii) If 6 more white balls are put in the bag, then

Total number of white balls = x + 6

Total number of balls in the bag = 12 + 6 = 18

$P(\text{White ball}) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}} = \frac{x+6}{12+6}$

According to the question,

Probability of drawing white ball in second case

= 2 × probability drawing of white ball in first case

$\Rightarrow \frac{x+6}{18} = 2\left(\frac{x}{12}\right)$

$\Rightarrow \frac{x+6}{18} = \frac{x}{6}$

$\Rightarrow 6x + 36 = 18x$

$\Rightarrow 12x = 36$

$\Rightarrow x = 3$

Hence, number of white balls = 3

Ex.12 What is the probability that a leap year, selected at random will contain 53 Sundays ?

Sol. Number of days in a leap year = 366 days

Now, 366 days = 52 weeks and 2 days

The remaining two days can be

(i) Sunday and Monday

(ii) Monday and Tuesday

(iii) Tuesday and Wednesday

(iv) Wednesday and Thursday

(v) Thursday and Friday

(vi) Friday and Saturday

(vii) Saturday and Sunday

For the leap year to contain 53 Sundays, last two days are either Sunday and Monday or Saturday and Sunday.

Number of such favourable outcomes = 2

Total number of possible outcomes = 7

$$\text{P(a leap year contains 53 sundays)} = \frac{2}{7}$$

Ex.13 Three unbiased coins are tossed together. Find the probability of getting :

(i) All heads,

(ii) Two heads

(iii) One head

(iv) At least two heads.

Sol. Elementary events associated to random experiment of tossing three coins are

HHH, HHT, HTH, THH, HTT, THT, TTH, TTT

$\therefore$ Total number of elementary events = 8.

(i) The event "Getting all heads" is said to occur, if the elementary event HHH occurs i.e. HHH is an outcome. Therefore,

$\therefore$ Favourable number of elementary events = 1

Hence, required probability = $\frac{1}{8}$

(ii) The event "Getting two heads" will occur, if one of the elementary events HHT, THH, HTH occurs.

$\therefore$ Favourable number of elementary events = 3

Hence, required probability = $\frac{3}{8}$

(iii) The events of getting one head, when three coins are tossed together, occurs if one of the elementary events HTT, THT, TTH happens.

$\therefore$ Favourable number of elementary events = 3

Hence, required probability = $\frac{3}{8}$

(iv) If any of the elementary events HHH, HHT, HTH and THH is an outcome, then we say that the event "Getting at least two heads" occurs.

$\therefore$ Favourable number of elementary events = 4

Hence, required probability = $\frac{4}{8} = \frac{1}{2}$.

Ex.14 A piggy bank contains hundred 50 p coins, fifty Re 1 coins, twenty Rs 2 coins and ten Rs 5 coins. If it is equally likely that one of the coins

will fall out when the bank is turned upside down, what is the probability that the coin

(i) will be a 50 p coin ?

(ii) will not be a ₹ 5 coin ?

Sol. Number of ₹50 coins = 100

Number of ₹1 coins = 50

Number of ₹2 coins = 20

Number of ₹5 coins = 10

(i) The number of favourable outcomes of 50 p coin to fall = 100

Total number of coins = 100 + 50 + 20 + 10 = 180

Total number of possible outcomes = 180

$$P = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$P\ (50\text{ p}) = \frac{100}{180} = \frac{5}{9}$$

(ii) Number of favourable outcomes of 5 Rs coin to not fall = 180 – 10 = 170

$$P = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$P\ (\text{not Rs. }5) = \frac{170}{180} = \frac{17}{18}$$

Ex.15 A box contains 20 balls bearing numbers, 1, 2, 3, 4, ... 20. A ball is drawn at random from the box. What is the probability that the number on the balls is

(i) An odd number

(ii) Divisible by 2 or 3

(iii) Prime number

(iv) Not divisible by 10

Sol. Total number of possible outcomes = 20

$$\text{Probability} = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

(i) Number of odds out of first 20 numbers = 10

Favourable outcomes by odd = 10

$$P(\text{odds}) = \frac{\text{Favourable outcomes of odd}}{\text{Total number of possible outcomes}} = \frac{10}{20} = \frac{1}{2}$$

(ii) The numbers divisible by 2 or 3 are 2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20.

Favourable outcomes of numbers divisible by 2 or 3 = 13

P (numbers divisible by 2 or 3)

$$= \frac{\text{Favourable outcomes of divisible by 2 or 3}}{\text{Total number of possible outcomes}} = \frac{13}{20}$$

(iii) Prime numbers out of first 20 numbers are 2, 3, 5, 7, 11, 13, 17, 19

Favourable outcomes of primes = 8

$$P(\text{primes}) = \frac{\text{Favourable outcomes of primes}}{\text{Total number of possible outcomes}} = \frac{8}{20} = \frac{2}{5}$$

(iv) Numbers not divisible by 10 are 1, 2, .. 9, 11, ...19

Favourable outcomes of not divisible by 10 = 18

P(not divisible by 10)

$$= \frac{\text{Favourable outcomes of not divisible by 10}}{\text{Total number of possible outcomes}} = \frac{18}{20} = \frac{9}{10}$$