STATISTICS

MEASURE OF CENTRAL TENDENCY (MEAN)

MEASURES OF CENTRAL TENDENCY

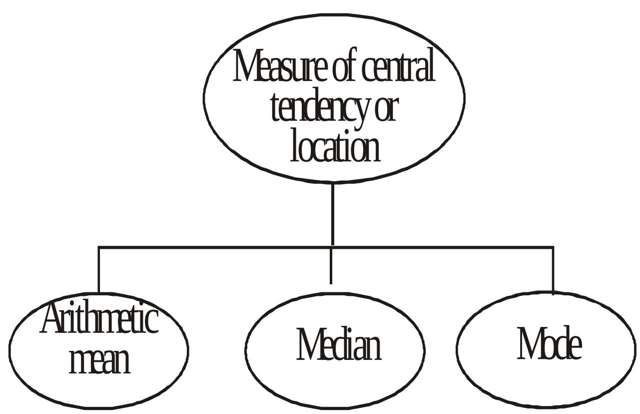
If the data is very large, the user cannot get much information from these data or its associated frequency distribution. In this case the information contained in data are represented by some numerical values, called averages. These averages are also called measures of central tendency or measures of location because they also give an idea about the concentration of the values in the central part of the distribution of data which describes the characteristics of the entire data or its associated frequency distribution.

The most commonly used averages are

(a) Arithmetic mean simply called mean

(b) Median and

(c) Mode.



Use of Summation (Σ) Notation

The symbol Σ (read : sigma) means summation. If $x_1, x_2, x_3, \ldots\ldots\ldots\ldots\ldots\ldots x_n$ are the 'n' values of a variable 'x' then their sum $x_1 + x_2 + x_3 + \ldots\ldots\ldots\ldots + x_n$ is denoted by $\sum_{i=1}^{n} x_i$ or simply $\sum x$.

Similarly the sum $k_1x_1 + k_2x_2 + \dots\dots\dots + k_nx_n$ is denoted by $\sum_{i=1}^{n} k_i x_i$ or simply $\sum kx$

Note: that $\sum_{i=1}^{n}(ax_i + b) = a\sum_{i=1}^{n} x_i + nb$

1 MEAN:

Arithmetic Mean of Individual Observation (Ungrouped Data):

If $x_1, x_2, x_3, \dots\dots\dots x_n$ are n values of a variable X, then the arithmetic mean or simply the mean of these values is denoted by $\overline{X}$ and is defined as

$$\overline{X} = \frac{x_1 + x_2 + x_3 + \dots\dots + x_n}{n} = \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)$$

Here, the symbol $\left(\sum_{i=1}^{n} x_i\right)$ denotes the sum $x_1, x_2, x_3, \dots\dots\dots x_n$

in other words, the arithmetic mean of a set of observations is equal to their sum divided by total number of observations.

Arithmetic Mean of Grouped Data (Discrete Frequency Distribution):

In a discrete frequency distribution the arithmetic mean may be computed by any one of the following methods:

(i) Direct Method

(ii) Short – cut Method

(iii) Step Deviation Method

We shall learn how to compute arithmetic mean by direct method only.

DIRECT METHOD:

Definition: If a variate X takes value $x_1, x_2, \dots\dots\dots x_n$ with corresponding frequencies $f_1, f_2, f_3, \dots\dots\dots f_n$ respectively, then arithmetic mean of these values is

$$\overline{X} = \frac{x_1f_1 + x_2f_2 + x_3f_3 + \dots\dots + x_nf_n}{f_1 + f_2 + f_3 + \dots\dots + f_n} \text{ or } \overline{X} = \frac{\sum_{i=1}^{n} x_i f_i}{N}$$

where $N = \sum_{i=1}^{n} f_i = f_1 + f_2 + f_3 + ... + f_n$

We use the following steps to compute arithmetic mean by direct method.

Step 1 Prepare the frequency table in such a way that its first column consists of the values of the variate and the second column the corresponding frequencies.

Step 2 Multiply the frequency of each row with the corresponding values of variable to obtain third column containing $f_i\ x_i$.

Step 3 Find the sum of all entries in III column to obtain $\sum x_i f_i$

Step 4 Find the sum of all the frequencies in II column to obtain $\sum f_i$ or N.

Step 5 Use the formula: $\overline{X} = \frac{\sum x_i f_i}{N}$

Ex.1 Find the mean of the following distribution.

x	3	7	8	11	15
f	5	10	10	7	8

Sol.

x_i	f_i	$x_i f_i$
3	5	15
7	10	70
8	10	80
11	7	77
15	8	120
	$\sum f_i = 40$	$\sum x_i f_i = 362$

Mean = $\overline{X} = \frac{\sum x_i f_i}{\sum f_i} = \frac{362}{40} = 9.05$

Ex.2 Find the mean of the first six multiples of 3.

Sol. The first six multiple of 3 are 3, 6, 9, 12, 15 and 18.

$$Mean = \frac{(3+6+9+12+15+18)}{6} = \frac{63}{6} = 10.5$$

Ex.3 The mean of 6, 10, x and 12 is 8. Find the value of x.

Sol. $$\overline{X} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{6+10+x+12}{4} = \frac{28+x}{4}$$

$8 = \frac{28+x}{4}$ $(\bar{X} = 8)$

$28 + x = 32$

$x = 4$, value of x is 4.

Ex.4 If the heights of 5 persons are 144 cm, 152 cm, 151 cm, 158 cm and 155 cm respectively. Find the mean height.

Sol. $$Mean\ height = \frac{114+152+151+158+155}{5} = \frac{760}{5} = 152 cm$$

Properties of Arithmetic Mean

1. The algebraic sum of the deviations of all the observations from their mean $(\bar{X})$ is zero. That is

$$(x_1 - x) + (x_2 - x) + \ldots + (x_n - x) = 0$$

$$x = \frac{1}{n}\sum_{i=1}^{n} x_i$$

2. If each observation is replaced by , the sum of all the observations remains unaltered.

3. If each observation is increased or decreased by a given constant, the arithmetic mean is also increased or decreased by the same constant.

4. If each observation multiplied or divided by a non zero constant, the mean is multiplied or divided by the same constant.

5. Combined Mean are the means of m groups with n_1, n_2,..... n_m as their number of observations, then combined mean x of all the groups taken together is given by

$$\bar{X}=\frac{n_1x_1+n_2x_2+\ldots+n_mx_m}{n_1+n_2+\ldots+n_m}$$

$$\bar{X}=\frac{\sum_{i=1}^{m} n_i x_i}{\sum_{i=1}^{m} n_i}$$

6. Arithmetic mean is affected by extreme values

Ex.5 The mean of 10 numbers is 20. If 5 is subtracted from every number, what will be the new mean?

Sol. Let x_1, x_2 .... x_{10} be 10 numbers with their mean equal to 20. Then.

$$\bar{X}=\frac{1}{n}\left(\sum x_i\right)$$

$$20=\frac{x_1+x_2+x_3+......+x_{10}}{10}$$

$x_1 + x_2 + ... + x_{10} = 200$

New numbers are $x_i - 5$, $x_2 - 5$, ... $x_{10} - 5$. Let be the mean of new numbers. Then

$$\bar{X}=\frac{(x_1-5)+(x_2-5)+.......+(x_{10}-5)}{10}$$

$$\bar{X}=\frac{200-50}{10}$$

$$\bar{X}=15$$

Ex.6 The mean of 100 items was found to be 30. It at the time of calculation two items were wrongly taken as 32 and 12 instead of 23 and 11, find the correct mean.

Sol. $n=100$, $\bar{X}=30$

$$\bar{X}=\frac{1}{n}\left(\sum x_i\right) \Rightarrow \sum x_i = n\bar{X}$$

$= 100 \times 30 = 3000$

incorrect value of $\sum x_i = 3000$

Now correct value of $\sum x_i$

$=$ incorrect value of $\sum x_i$ – (sum of incorrect value + sum of correct value)

Correct value of $\sum x_i$ $= 3000 - (32 + 12) + (23 + 11) = 2990$

$$Correct\ mean = \frac{Correct\ value\ of\ \sum x_i}{n} = \frac{2990}{100} = 29.9$$