SURFACE AREAS AND VOLUMES

SPHERE AND HEMISPHERE

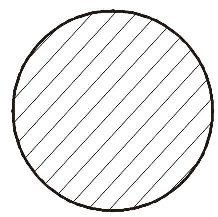
SPHERE :

Objects like football, volleyball, etc. are said to have the shape of a sphere. A sphere is the collection of points in space, equidistant from a fixed point.

The fixed point is called the centre and fixed distance is called the radius of sphere. A segment joining two points on the sphere and passing through its centre is called diameter of the sphere.

A sphere is a solid generated by the revolution of a semicircle about its diameter. The centre and radius of the semicircle are respectively the centre and radius of the sphere.

Formulae:



(i) Surface area of a sphere = $4\pi r^2$ sq units

(ii) Volume of a sphere = $\frac{4}{3}\pi r^3$ cu units

(iii) Curved surface area of a hemisphere = $2\pi r^2$ sq units

(iv) Total surface area of a hemisphere = $2\pi r^2 + \pi r^2 = 3\pi r^2$ sq units

(v) Volume of a hemisphre = $\frac{2}{3}\pi r^3$ cu units.

Ex.1 A spherical ball of lead 3cm in diameter is melted and recast into three spherical balls. The diameters of two of these are 1 cm and 1.5cm. Find the diameter of the third ball.

Sol. It is given that

The diameter of a spherical ball = 3cm

$\Rightarrow$ radius of it = 1.5 cm = $\frac{3}{2}$ cm

So, volume of it = $\left[\frac{4}{3}\pi\times\left(\frac{3}{2}\right)^3\right]$ cm^3

$= \left(\frac{4}{3}\pi \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2}\right) cm^3 = \frac{108\pi}{24} cm^3 = \frac{9\pi}{2} cm^2$

This spherical ball is melted and recast into three small spherical balls. The diameters of two of these are 1 cm and 1.5 cm respectively.

So, volume of the two spherical balls.

$= \left[\frac{4}{3}\pi \times \left\{\left(\frac{1}{2}\right)^3 + \left(\frac{3}{4}\right)^3\right\}\right] cm^3$

$= \left[\frac{4}{3}\pi\left(\frac{1}{8} + \frac{27}{64}\right)\right] cm^3$

$= \left(\frac{4}{3}\pi \times \frac{35}{64}\right) cm^3 = \frac{140\pi}{192} cm^3$

Let r be the radius of the third small spherical ball.

Thus, volume of the third ball = volume of the big spherical ball – sum of volume of two small spherical balls.

$\Rightarrow \frac{4}{3}\pi r^3 = \frac{9\pi}{2} - \frac{140\pi}{192}$

$\Rightarrow \frac{4}{3} r^3 = \frac{9}{2} - \frac{140}{192} = \frac{864 - 140}{192} = \frac{724}{192}$

$\Rightarrow r^3 = \frac{724 \times 3}{4 \times 192} = \frac{181}{64} \quad \Rightarrow r = \sqrt[3]{\frac{181}{64}} cm$

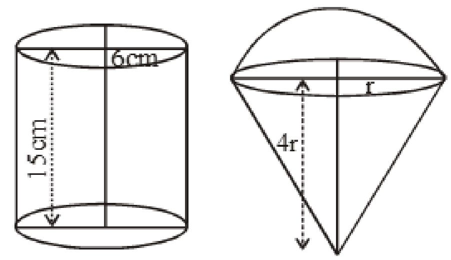
Hence, diameter of the 3rd spherical ball = 2r

$= 2 \times \frac{\sqrt[3]{181}}{\sqrt[3]{64}} = 2 \times \frac{\sqrt[3]{181}}{4} = \frac{\sqrt[3]{181}}{2} cm$

Ex.2 A cylindrical container is filled with ice-cream whose diameter and the height are 12cm and 15cm respectively. The whole ice-cream is distributed to 10 children in equal inverted cones having hemispherical tops. Find the diameter of the ice-cream, if the height of the conical part is twice the diameter of its base.

Sol. We have radius of the cylindrical container = r = $\frac{12}{2}$ = 6cm and

height of it (h) = 15cm.



So, its volume = $\pi r^2 h = (\pi \times 6^2 \times 15)$ cm^3

= $(\pi \times 36 \times 15)$ cm^3 = 540 πcm^3

Let radius of the hemispherical part of the ice-cream = radius of the base of the conical part of the ice-cream = r

So, height of the conical part of the icecream = 4r

So, the volume of one ice-cream = volume of the hemispherical part + volume of the conical part.

$= \left(\frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 \times 4r\right)$cm^3

$= \left(\frac{2}{3}\pi r^3 + \frac{4}{3}\pi r^3\right)$cm^3 = $(2\pi r^3)$ cm^3

Volume of 10 ice-cream = $(10 \times 2\pi r^3)$cm^3 = $(20\pi r^3)$ cm^3

Here, volume of 10 ice-cream = volume of the cylindrical container

$\Rightarrow\ 20\pi r^3 = 540\pi \Rightarrow 20r^3 = 540$

$\Rightarrow\ r^3 = \frac{540}{20} = 27 \Rightarrow r = \sqrt[3]{27} = 3$ cm

Hence, the required diameter of the ice-cream = 2r = 2 × 3 = 6cm

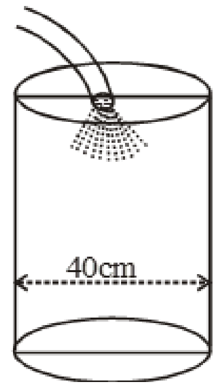
Ex.3 Water flows out through a circular pipe, whose internal diameter is 2cm, at the rate of 0.7m/sec into a cylindrical tank, the radius of whose base is 40cm. By how much will the level of water rise in half an hour ?

Sol. We have volume of water flows out through a circular pipe in 1 second = volume of a cylinder of the base of radius 1 cm ($r = \frac{2}{2} = 1$ cm) and

height 70 cm (h = 0.7m = 70 cm)

$= \pi r^2 h = \left(\frac{22}{7} \times 1^2 \times 70\right)$ cm^3 = 220 cm^3

So, volume of water passed through the pipe into the cylindrical tank in 1800 seconds $\left(\frac{1}{2}\text{hour} = \frac{3600}{2} = 1800\text{sec}\right)$



$= (220 \times 1800)\ \text{cm}^3 = 396000\ \text{cm}^3$

Thus, rise in the level of water in 1800 sec or half an hour

$= \frac{\text{Total volume of water poured into the cylindrical tank}}{\text{Area of base of the cylindrical tank}}$

$= \frac{396000\ \text{cm}^3}{\pi \times 40^2\ \text{cm}^2}$ (radius of the base of the cylindrical tank = 40 cm)

$= \left(\frac{396000\ \text{cm}^3}{\frac{22 \times 1600}{7}\ \text{cm}^2}\right) = \left(\frac{396000 \times 7}{22 \times 1600}\right)\text{cm}$

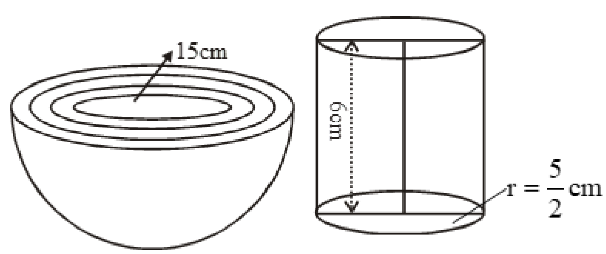
$= 78.75\ \text{cm} \cong 79\ \text{cm}$

Hence, water rise upto 79 cm in half an hour

Ex.4 A hemispherical bowl of internal radius 15cm is full of a liquid. The liquid is to be filled into some bottles of cylindrical in shape whose diameters and heights are 5 cm and 6 cm respectively. Find the number of bottles necessary to empty the bowl.

Sol. We have internal radius of the hemispherical bowl = R = 15 cm.

So, its volume = $\frac{2}{3}\pi \times R^3$



$= \left[\frac{2}{3} \times \pi \times (15)^3\right]\text{cm}^3 = \left(\frac{2}{3} \times \pi \times 15 \times 15 \times 15\right)\text{cm}^3$

$= 10 \times 15 \times 15\ \pi\text{cm}^3 = 2250\ \pi\text{cm}^3$

So, volume of the entire liquid = $2250\ \pi\text{cm}^3$

The liquid is to be filled into some bottles of cylindrical in shape whose diameters and heights are 5 cm and 6 cm respectively. So, radius of the cylindrical bottle = $\frac{5}{2}$ cm and height of it = 6cm

So, volume of one cylindrical bottle = $\pi r^2 h$

$$= \left(\pi\times\frac{5}{2}\times\frac{5}{2}\times 6\right)\text{cm}^3 = \left(\frac{75\pi}{2}\right)\text{cm}^3$$

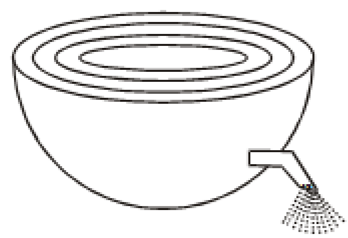
So, the number of bottles necessary to empty the hemispherical bowl

$$= \frac{\text{Volume of the entire liquid in the bowl}}{\text{Volume of one cylindrical bottle}}$$

$$= \frac{2250\ \text{cm}^3}{\frac{75\pi}{2}\ \text{cm}^3} = \frac{2250\times 2}{75} = 60$$

Ex.5 A hemispherical tank of radius $1\frac{3}{4}$ m is full of water. It is connected with a pipe which empties it at the rate of 7 lt/sec. How much time will it take to empty the tank completely ?

Sol. We have radius of the hemispherical tank



$= 1\frac{3}{4} = \frac{7}{4}$ m. It is full of water.

So, volume of entire water in the hemispherical tank

$$= \left[\frac{4}{3}\pi\times\left(\frac{7}{4}\right)^3\right]\text{m}^3 = \left(\frac{4}{3}\times\frac{22}{7}\times\frac{7}{4}\times\frac{7}{4}\times\frac{7}{4}\right)\text{m}^3$$

This tank is connected with a pipe which empties it at the rate of 7lt/sec.

So, volume of water flows out in 1 sec = 7 litre

$= (7 \times 1000)\text{cm}^3 = 7000\ \text{cm}^3$

$$= \left(\frac{7000}{100\times 100\times 100}\right)\text{m}^3$$

Thus, total time will be taken to empty the tank full of water

$$= \left(\frac{4}{3}\times\frac{22}{7}\times\frac{7}{4}\times\frac{7}{4}\times\frac{7}{4}\right) \div \left(\frac{7000}{100\times 100\times 100}\right)$$

$$= \left(\frac{22\times 49}{48}\div\frac{7}{1000}\right)\text{sec}$$

$$= \frac{22\times 49\times 1000}{48\times 7}\text{sec} = \frac{19250}{12}\text{sec}$$

$= \left(\frac{19250}{12 \times 60}\right) \min = \frac{1925}{72} \min = 26.73 \text{ minutes}$

Hence, the required time is 26.73 minutes.

Ex.6 A hemispherical bowl of internal radius 9cm is full of liquid. This liquid is to be filled into cylindrical shaped small bottles each of diameter 3 cm and height 4 cm. How many bottles are necessary to empty the bowl ?

Sol. Volume of the hemispherical bowl = $\frac{2}{3}\pi R^3 = \frac{2}{3} \times \frac{22}{7} \times (9)^3$

(R = Internal radius of the hemispherical bowl = 9 cm)

$= \left(\frac{2}{3} \times \frac{22}{7} \times 9 \times 9 \times 9\right) \text{cm}^3$

So, volume of the liquid in the bowl = $\left(\frac{2}{3} \times \frac{22}{7} \times 9 \times 9 \times 9\right) \text{cm}^3$

Volume of a bottle = $\pi r^2 h = \frac{22}{7} \times \left(\frac{3}{2}\right)^2 \times 4$

(r = radius of the cylindrical bottle = $\frac{3}{2}$ cm and height (h) = 4 cm)

$= \frac{22}{7} \times \frac{9}{4} \times 4 = \frac{198}{7} \text{cm}^3.$

Number of bottles required to empty the bowl

$= \frac{\text{Volume of the liquid in the bowl}}{\text{volume of the one bottle}}$

$= \frac{2}{3} \times \frac{22}{7} \times 9 \times 9 \times 9 \div \frac{198}{7}$

$= \frac{2}{3} \times \frac{22}{7} \times 9 \times 9 \times 9 \times \frac{7}{198} = 54$

Hence, the required number of bottle necessary to empty the bowl is 54.

Ex.7 Water in a canal, 30 dm wide and 12 dm deep is flowing with velocity of 10 km per hour. How much area will it irrigate in 30 minutes, if 8cm of standing water is required for irrigation ?

Sol. We have

$30\text{ dm} = \frac{30}{10}\text{m}, 12\text{dm} = \frac{12}{10}\text{m}$

$10\text{ km} = 10 \times 1000\text{ m}$

Volume of water flowing in canal in 1 hour

$= \frac{30}{10} \times \frac{12}{10} \times 10 \times 1000 = 36000\text{ m}^3.$

Volume of water flowing in canal in 30 minutes

$= \left(\frac{1}{2}\text{hour}\right) = \frac{36000}{2} = 18000\text{ m}^3.$

Then Area that will be irrigated in $\frac{1}{2}$ hour

$= \frac{\text{volume}}{\text{height}} = \frac{18000\text{ m}^3}{8\text{ cm}}$

$= \left(\frac{18000 \times 100}{8}\right)\text{m}^2 = 225000\text{ m}^2$

Hence, the required amount of standing water needed is 225000 m^2.