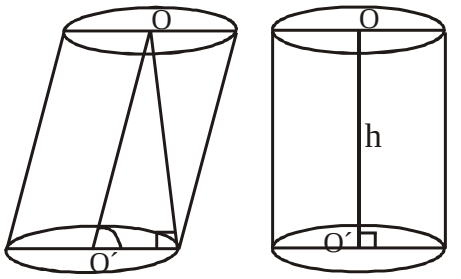
SURFACE AREAS AND VOLUMES

CYLINDER

RIGHT CIRCULAR CYLINDER :

A cylinder is a solid with curved surface (lateral surface) and congruent circular ends. In a right circular cylinder, the line segment joining the centres (O and O′) of the bases is perpendicular to the two bases. The line OO′ is called the axis of the cylinder. A right circular cylinder is defined as a solid generated by the revolution of rectangle about one of its sides.



Not a right circular cylinder

Right circular cylinder

(i) Curved surface area of a cylinder

Radius of bases = r, height = h,

Curved surface area = $2\pi rh$

(ii) Total surface area of a closed cylinder

$= 2\pi r^2 + 2\pi rh = 2\pi r\,(r+h)$

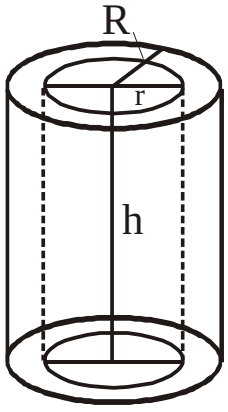
(iii) Total surface area of a cylinder open at one end

$= \pi r^2 + 2\pi rh$

$= \pi r(r + 2h)$

(iv) Volume of a cylinder = Area of base × height = $\pi r^2 h$

HOLLOW CYLINDER :



Outer radius = R, Inner radius = r, height = h

(a) Outer curved surface area = $2\pi Rh$

(b) Inner curved surface area = $2\pi rh$

(c) Total curved surface area (outer + inner) = $2\pi h(R + r)$

(d) Area of a ring = $\pi(R^2 - r^2)$

(e) Total surface area of a hollow cylinder

$= 2\pi h\,(R + r) + 2\pi(R^2 - r^2)$

$= 2\pi h\,(R + r) + 2\pi(R + r)\,(R - r)$

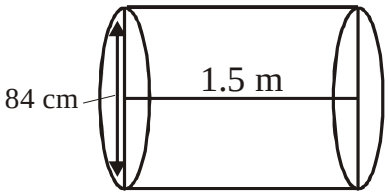
$= 2\pi\,(R + r)\,(h + R - r)$

(f) Volume of the hollow cylinder

$= \pi(R^2 - r^2)h$

$= \pi h(R + r)\,(R - r)$

Ex.1 The diameter of a road roller 1.5 m long is 84 cm. If it takes 100 revolution to level the ground, find the cost of levelling this ground at the rate of Rs. 0.50 per m^2.



Sol Radius of the road roller $= 84/2 = 42$ cm

Length of the roller $= 1.5$ m

Area levelled in one revolution $= 2\pi rh$

$= 2 \times 22/7 \times 0.42 \times 1.5 = 3.96\ m^2$

$\therefore$ Area levelled in 100 revolutions $= 3.96 \times 100 = 396\ m^2$

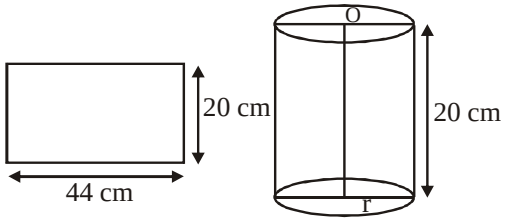
Rate of levelling the ground = Rs. 0.50 per m^2

$\therefore$ Cost of levelling the ground = Rs. 396×0.50 = Rs. 198

Hence, cost of levelling the ground is Rs. 198

Ex. 2 A rectangular sheet of aluminium foil is 44 cm long and 20 cm wide. A cylinder is made by rolling the foil along its length. Find the volume of the cylinder so formed.

Hint:



Sol. $22/7 \times 7 \times 7 \times 20 = 3080\ cm^3$

Ex.3 A cylindrical vessel, without lid, has to be tin coated including both of its sides. If the radius of its base is $\frac{1}{2}$ m and its height is 1.4 m, calculate the cost of tin-coating at the rate of Rs. 50 per 1000 cm^2.

Sol. Radius of the base (r) $= \frac{1}{2}$m

$= \frac{1}{2} \times 100 \text{ cm} = 50 \text{ cm}$

Height (h) $= 1.4$ m

$= 1.4 \times 100$ cm

$= 140$ cm.

Surface area of to tin-coated $= 2\ (2\pi r + \pi r^2)$

$= 2[2 \times 3.14 \times 50 \times 140 + 3.14 \times (50)^2]$

$= 2[43960 + 7850] = 2(51810) = 103620 \text{ cm}^2$

$\therefore$ Cost of tin-coating at the rate or Rs. 50 per 1000 cm^2

$= \frac{50}{1000} \times 103620 = \text{Rs } 5181.$ **Ans.**

Ex.4 The diameter of a roller 120 cm long is 84 cm. If its takes 500 complete revolutions to level a playground determine the cost of leveling at the rate of Rs. 25 per square metre. (Use $\pi = \frac{22}{7}$)

Sol. $2r = 84$ cm

$\therefore \quad r = \frac{84}{2} \text{ cm} = 42 \text{ cm}$

$h = 120$ cm

Area of the playground leveled in one complete revolution $= 2\pi rh$

$= 2 \times \frac{22}{7} \times 42 \times 120 \times 31680 \text{ cm}^2$

$\therefore$ Area of the playground $= 31680 \times 500 \text{ cm}^2$

$= \frac{31680 \times 500}{100 \times 100} \text{ m}^2 = 1584 \text{ m}^2$

$\therefore$ Cost of leveling @ Rs 25 per square metre = Rs $1584 \times 25 = 39600$.

Ex.5 Water flows at the rate of 10 per minute through a cylindrical pipe having its diameter as 5 mm. How much time will it take to fill a conical vessel whose diameter of the base is 40 cm and depth 24 cm ?

Sol. Diameter of the pipe = 5 mm = $\frac{5}{10}\text{cm} = \frac{1}{2}\text{cm}$

$\therefore$ Radius of the pipe = $\frac{1}{2} \times \frac{1}{2}\text{cm} = \frac{1}{4}$ cm.

In 1 minute, the length of the water column in the cylindrical pipe

= 10 m = 1000 cm.

$\therefore$ Volume of water that flows out of the pipe in 1 minute $= \pi \times \frac{1}{4} \times \frac{1}{4} \times 1000\ \text{cm}^3$.

Also, volume of the cone $= \times\ \frac{1}{3} \times \pi \times 20 \times 20 \times 24\ \text{cm}^3$.

Hence, the time needed to fill up this conical vessel $= \left(\frac{1}{3}\pi \times 20 \times 20 \times 24 \div \pi \times \frac{1}{4} \times \frac{1}{4} \times 1000\right)$

minutes $= \left(\frac{20 \times 20 \times 24}{3} \times \frac{4 \times 4}{100}\right) = \frac{4 \times 24 \times 16}{30}$ minutes $= \frac{256}{5}$ minutes = 51.2 minutes.

Hence, the required time is 51.2 minutes. **Ans.**