SURFACE AREAS AND VOLUMES

CUBOID AND CUBE

INTRODUCTION :

Mensuration is a branch of mathematics which concerns with the measurement of lengths, area and volume of the plane and solid figures.

SOLIDS :

A plane figure may have one dimension or two dimensions. Triangles and quadrilaterals have two dimensions. For two-dimensional figures, the dimensions are length and breadth or width or height. But many objects such as a brick, a match box, a pencil, a marble, a tank, an ice cream cone etc., have a third dimension.

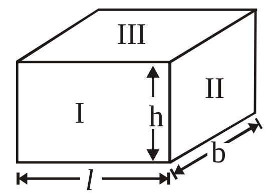
Thus, a solid is a three dimensional object. In general, any object occupying space can be called a solid. Some solids like prisms, cubes and cuboids etc. have plane or flate surface while some solids like cone, cylinder etc. have curved surfaces as well as flat surfaces. Spheres have only curved surfaces. Lateral surface area of a solid having a curved surface is referred to as curved surface area.

The amount of space enclosed by the bounding surface or surfaces of a solid is called the volume of the solid and is measured in cubic units.

CUBOID :

A solid like an ordinary brick, each face of which is rectangle is called a rectangular solid or cuboid.

A cuboid has 6 rectangular plane surfaces, called faces. It has 12 edges and 8 vertices. A cuboid is also called a rectangular parallelopiped. If a cuboid is l cm long b cm broad and h cm high, then



(i) Volume of cuboid = (area of base) × height

$= (l \times b) \times h$

$= lbh\ cm^3$

(ii) Total Surface Area of Cuboid :

We know that the outer surface of cuboid is made up of six rectangles and the area of rectangle can be found by multiplying length by breadth.

Area of I rectangle = $l \times h$

Area of opposite of I rectangle = $l \times h$

Area of II rectangle = $b \times h$

Area of opposite of II rectangle = $b \times h$

Area of III rectangle = $l \times b$

Area of opposite of III rectangle = $l \times b$

Surface Area of a cuboid = Area of 6 rectangles

$= lb + lb + bh + bh + hl + hl$

$= 2lb + 2bh + 2hl = 2(lb + bh + hl)$

Total surface area of a cuboid = $2(lb + bh + hl)$

(iii) Length of the diagonal of cuboid = cm

(iv) Lateral surface area of cuboid = $2(l + b) \times h\ cm^2$

(v) If the areas of three adjacent faces of a cuboid are x, y, z and its volume is V then $V^2 = xyz$

Ex.1 A right circular cylinder which has a height of 21 cm and the base radius of 5 cm. Find the curved surface area of the cylinder.

Sol. Height of the cylinder, h = 21 cm.

Base radius, r = 5 cm

Curved surface area of the cylinder = $2\pi rh = \times 5 \times 21 cm^2 = 660\ cm^2$

Ex.2 The height of a cylinder is 15 cm. The curved surface area is 660 sq. cm. Find its radius. (Take π as $\frac{22}{7}$)

Sol. Height of the cylinder, h = 15 cm

Curved surface area = 660 sq. cm.

Curved surface area of the cylinder = $2\pi rh$ = 660 sq. cm

$\therefore \quad 2\pi rh = 660$ sq. cm

$\Rightarrow \quad 2 \times \frac{22}{7} \times r \times (15) = 660$

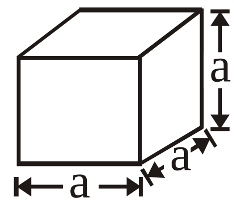
$\Rightarrow \quad r = 660 \times \frac{7}{22 \times 2 \times 15} = 7$ cm

$\therefore \quad$ Radius (r) = 7 cm

CUBE :

It is a particular case of a cuboid where length = breadth = height. Each edge of a cube is called its side.

Let a be the length of an edge of a cube, then



(i) Volume of a cube = a^3 cubic unit

(ii) Total Surface Area of Cube :

the total surface area of this cube would be

$2(a \times a + a \times a + a \times a)$ i.e., $6a^2$ giving us

Total surface area of cube = $6a^2$, where a is the edge of the cube.

(iii) Lateral surface area of a cube = $4a^2$ sq unit

(iv) Length of diagonal $= \sqrt{a^2 + a^2 + a^2} = \sqrt{3}a$

Ex.3 Three equal cubes are placed adjacently in a row. Find the ratio of the total surface area of the new cuboids to that of the sum of the surface areas of three cubes.

Sol. Let the side of each of the three equal cubes be a cm.

Then surface area of one cube = $6a^2$ cm^2

$\therefore$ Sum of the surface areas of three cubes = $3 \times 6a^2 = 18a^2$ cm^2

For new cuboids

length (ℓ) = 3a cm

breadth (b) = a cm

height (h) = a cm

$\therefore$ Total surface are of the new cuboids $= 2(\ell \times b + b \times h + h = \ell)$

$= 2[3a \times a + a \times a + a \times 3a]$

$= 2[3a^2 + a^2 + 3a^2] = 14a^2$ cm.2

$\therefore$ Required ratio $= \frac{\text{Total surface area of the new cuboid}}{\text{Sum of the surace areas of three cube}}$

$= \frac{14a^2}{18a^2} = \frac{7}{9} = 7:9$ **Ans.**

Ex.4 A class room is 7 m long, 6.5 m wide and 4 m high. It has one door 3 m × 1.4 m and three windows each measuring 2 m × 1 m The interior walls are to be colour-washed. The contractor charges Rs. 15 per sq. m. Find the cost of colour washing.

Sol. $\ell = 7$m, n = 6.5 m and h = 4 m

$\therefore$ Area of the room $= 2(\ell + b)h = 2(7 + 6.5)\ 4 = 108\ m^2$

Area of door $= 3 \times 1.4 = 4.2\ m^2$

Area of one window $= 3 \times 2 = 6\ m^2$

$\therefore$ Area of 3 windows $= 3 \times 2 = 6\ m^2$

$\therefore$ Area of the walls of the room to be colour washed $= 108 - (4.2 + 6)$

$= 108 - 10.2 = 97.8\ m^2$

$\therefore$ Cost of colour washing Rs. 15 per square metre = Rs. 97.8×15 = Rs. 1467.

Ex.5 The surface area of a cuboid is 1372 cm^2. If its dimensions are in the ratio 4 : 2 : 1, find them.

Sol. Let the length, breadth and height of cuboid are 4x, 2x andx.

Surface area of a cuboid = 2(lb + bh + lh)

$2(4x \times 2x + 2x \times x + x \times 4x) = 1372\ cm^2$

$2(8x^2 + 2x^2 + 4x^2) = 1372\ cm^2$

$2 \times 14x^2 = 1372$

$28x^2 = 1372$

$x^2 = 1372/28$

$x^2 = 49$

$x = 7$

length = 28 cm, breadth = 14 cm, height = 7 cm

Ex.6 Two cubes of side 2 cm each are joined end to end, find the volume of the cuboid so formed.

Sol. When two cubes of side 2 cm each are joined end to end then,

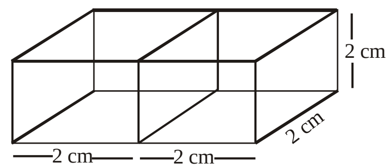
$l = (2 + 2)$ cm,

$b = 2$ cm

$h = 2$ cm

volume = $l \times b \times h$

$V = 4 \times 2 \times 2 = 16\ cm^3$



Ex.7 A rectangular water reservoir is 7.2 m by 2.5 m at the base. Water flows into it through a pipe whose cross-section is 5 cm × 3 cm at the rate of 10 m per second. Find the height to which the water will rise in the reservior in 40 minutes.

Sol. Area of the cross-section of the pipe = $(0.05 \times 0.03)\ m^2$

Volume of water that flows in 1 second = $(0.05 \times 0.03 \times 10)\ m^3$

Volume of water that flows in 40 minutes

$= 0.05 \times 0.03 \times 10 \times 60 \times 40\ m^3$

$= \frac{5 \times 3 \times 10 \times 60 \times 40}{100 \times 100} m^3$

$= 36\ m^3$

Volume of water in the tank = $36\ m^3$

Area of the base = $7.2 \times 2.5\ m^2 = 18\ m^2$

$\therefore$ Height = $\frac{36}{18} = 2m$

Ex.8 The dimensions of a cinema hall are 100 m, 50 m and 18m. How many persons can sit in the hall, if each required 150 m^3 of air ?

Sol. ℓ = 100 m

b = 50 m

h = 18 m

$\therefore$ Volume of the cinema hall = ℓbh

= 100 × 50 × 18 = 90000 m^3

Volume occupied by 1 person = 150 m^3

$\therefore$ Number of persons who can sit in the hall

$$= \frac{\text{Volume of the hall}}{\text{Volume occupied by 1 person}} = \frac{90000}{150} = 600$$

Hence 600 persons can sit in the hall. **Ans.**

Ex.9 The outer measurements of a closed wooden box are 42 cm, 30 m and 27 cm. If the box is made of 1 cm thick wood, determine the capacity of the box.

Sol. **Outer dimensions**

ℓ = 42 cm

b = 30 cm

h = 27 cm

Thickness of wood = 1 cm

Inner dimensions

ℓ = 42 - (1 + 1) = 40 cm

b = 30 - 1(1 + 1) = 28 cm

h = 27 - (1 + 1) = 25 cm

Capacity of the box $\ell \times b \times h$

$= 40 \times 28 \times 25 = 28000\ cm^2$. **Ans.**

Ex.10 If v is the volume of a cuboids of dimensions a,b, and c and s is its surface area, then prove that $\frac{1}{v} = \frac{2}{a}\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$

Sol. L.H.S. $= \frac{1}{v} = \frac{1}{abc}$ ....(i)

R.H.S. $= \frac{2}{s}\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$

$= \frac{2}{2(ab+bc+ca)}\left(\frac{bc+ca+ab}{abc}\right)$

$= \frac{1}{abc}$ ....(ii)

from (i) and (ii) $\frac{1}{v} = \frac{2}{s}\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$. **Hence Proved.**